Econ 110: Introduction to Economic Theory

10th Class **2/11/11**

go over practice problems

second of three lectures on producer theory

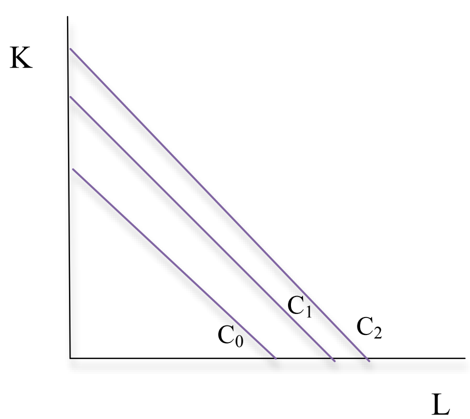
Last time we showed the first type of constraint operating on the firm: the tyranny of time, which decides what factors are variable and what technologies are available

Let's keep operating in the medium run for the time being, so the firm has two variable inputs, K and L, and a particular production function Q = f(K, L
Next constraint on the firm: the prices of capital and labor (per unit)

set up $P_K = r$ (rental rate of capital) and $P_L = w$ (wage) by convention

Then the cost C to the firm of purchasing inputs = rK + wL

For various "budget" levels to the firm, there are multiple ways to spend your budget between K and L. We can show these ways as <u>isocost lines:</u>



mark intercepts as C/r, C/w (amount you can buy if you spend all on one input)
along each line, the same amount of total cost accrues to the firm.
What is the slope of each isocost line?

rewrite the cost to be rK = C – wL, K = $K = \frac{C}{r} - \frac{w}{r}L$

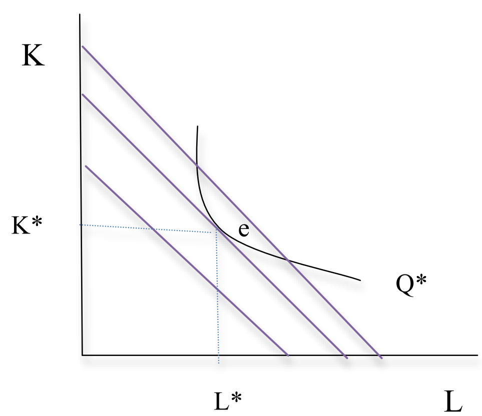
$$\frac{dK}{dL} = -\frac{w}{r}$$

you can probably see where we're going now with this in our parallel to consumer theory, namely that for the firm to be making an optimal choice, the slope of the isoquant (the MRTS) has to be equal to the slope of the isocost curve/line:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

If this condition does not hold, the firm could reduce costs further.

Consider the graphical representation of this condition:



note that in this case there is only one curved line (the isoquant) and a whole family of straight lines (the isocost curves) and the goal is to get to the lowest isocost curve possible while staying on the given isoquant (so kind of the flip of the consumer's problem).

Let's first break the producer's problem of how to maximize profit into two parts:

1) the cost minimization problem: what's the cheapest way of producing a given level of output Q*? You can think of this as the lower-level manager's problem; the directive comes down from

above regarding how much the firm is to produce, and the lower-level manager has to figure out how to do it as cheaply as possible

2) the output level choice. Think of this as the higher-level manager's problem

Let's solve the lower-level problem first. So assume that the desired output level is given (Q^*)

then the lower-level manager solves a two-equation in two unknown (K and L) system, where he takes w, r, and Q^* as given, where one equation is the one we just derived and the other is the production function for producing the given level of Q^*:

$$1)\ \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$2)\ Q^* = f(K,L)$$

the second equation basically tells the manager what isoquant he has to operate on, and then he picks the cost-minimizing point on that isoquant.

Assume you solve this system. Then you have two equations for K^* and L^*:

$$K^* = K(w, r, Q^*)$$
$$L^* = L(w, r, Q^*)$$

these are the input demand functions (again, note the parallel with the consumer's demand functions. consumers are like producers of utility and a utility function is like a production function). So as the factor prices vary and the amount of Q to be produced varies, the quantity demanded of K and L varies. Note we could calculate own-price and cross-price elasticities from these demand functions.

Plug these optimal amounts back into the cost function:

$$C = rK^* + wL^* = C(w, r, Q^*)$$

Holding w and r constant for the moment, then we can write the (total) cost curve for the firm as:

$TC = C(Q)$

so as Q varies, we can see how total cost varies, where it is always the minimum possible cost for producing the particular level of output Q (it's always possible to spend more; the trick is to spend as little as possible)

Now the higher-level manager can just work with this cost curve, resting secure in the belief that the lower-level manager will be able to get the lowest possible cost for whatever level of output Q the upper-level manager chooses.

Note that we can now define two new concepts:

<u>average cost</u> $AC = \frac{C}{Q}$

<u>marginal cost</u> $MC = \frac{dC}{dQ}$

Now we can state the upper-level manager's problem, which is the same as the firm's problem, as maximizing

$$\Pi = TR - TC$$

$$= PQ - C(Q)$$

To maximize Π by choosing the optimal level of output, set:

$\frac{d\Pi}{dQ} = 0$, so $P - \frac{dC}{cQ} = 0$, or P – MC = 0, or P = MC

And in general you can solve this single equation for the optimal level of output Q*

this is where the third constraint on the firm comes into play: takes demand conditions as given In the model I just presented, that reduces to the assumption that the firm takes (output) price as given (and equal to P)

So it turns out really the only choice the firm has in this model is what output to produce, Q

Let's continue our discussion of the different planning horizons

First let's divide up costs a little differently:

total cost (TC, or C) can also be divided into <u>total variable cost</u> (VC) and <u>total fixed cost</u> (FC), where fixed costs are those that do not vary with output level (give examples):

$$TC = VC + FC \quad (VC = rK + wL, \text{ so } TC = rK + wL + FC \text{ in our two-factor case})$$

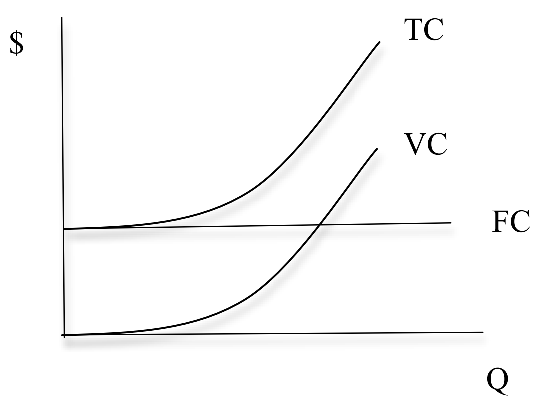
and we can calculate average cost (AC), <u>average variable cost</u> (AVC), and <u>average fixed cost</u> (AFC) by dividing that equation through by Q:

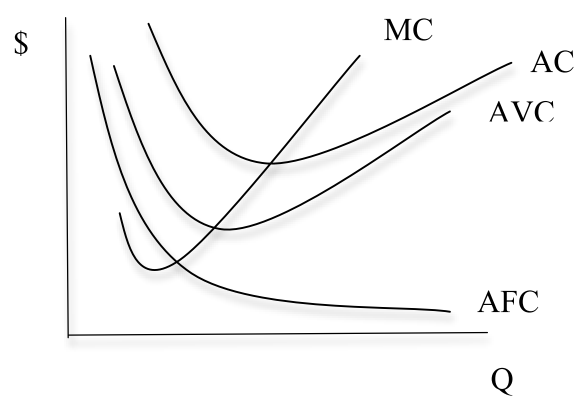
$$\frac{TC}{Q} = \frac{VC}{Q} + \frac{FC}{Q} \quad \text{, so } AC = AVC + AFC$$

and finally, note that since by definition fixed costs do not vary with output level:

$$\frac{dTC}{dQ} = \frac{dVC}{dQ} = MC$$

diagrams of total and average&marginal cost curves:





Professor Joyce Jacobsen

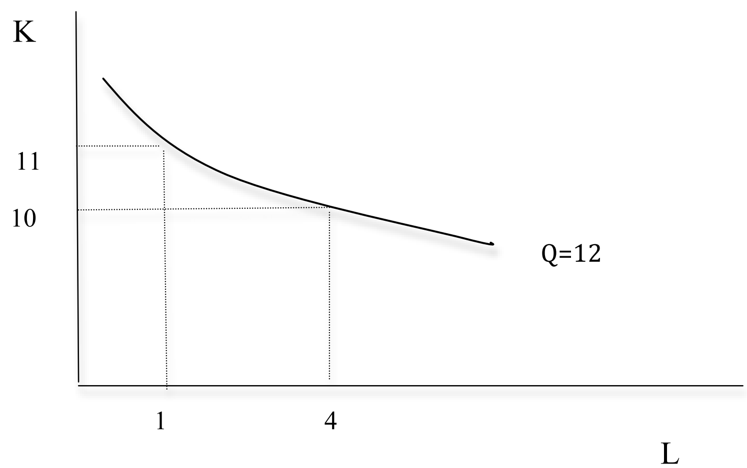
Economics 110-01
Spring Semester 2010-11

Answers to Practice Problems from 2/9/11

I. 1) $MP_K = 1$; $MP_L = \frac{1}{2\sqrt{L}} = \frac{1}{2}(L)^{-\frac{1}{2}}$

2) $Q = 2 + 2 = 4$; $MP_K = 1$; $MP_L = \frac{1}{4}$

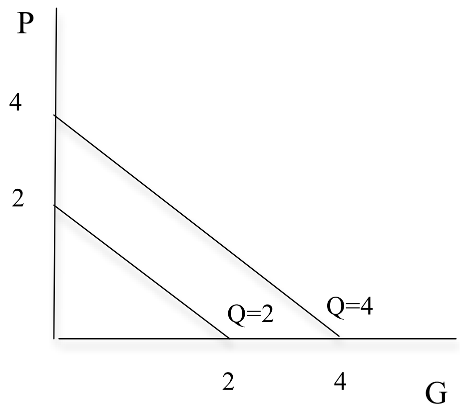
3) e.g., K=10 and L = 4; K = 11 and L = 1



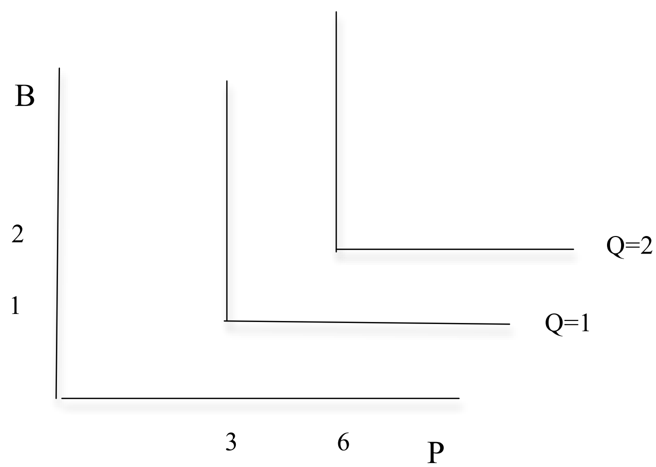
4) $MRTS = 2\sqrt{L}$

II.

1)



2)



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Practice Problems 2/11/11

I. Consider the following production function:

$$Q = K + \sqrt{L}$$

1) If r = 2 and w = 1, what is the slope of an isocost line?

2) If the firm wants to produce Q = 21, what are the optimal quantities of labor and capital?

3) What is the firm's cost of producing Q = 21?

II. In each of the following cases, sketch a couple of isocost curves and an isoquant on the same diagram and indicate where the optimal point is for the producer:

1) An isoquant for making paper airplanes out of pink paper and green paper; pink paper costs 1 cent a sheet and green paper costs 2 cents a sheet.

2) An isoquant for making triple burgers (one bun and three patties per burger); one bun costs the same as one patty.