do the love song for economists in honor of valentine’s day
(couldn’t get it to load fast enough for class, but feel free to enjoy it on your own)

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go over answers to practice problems

last of three lectures on producer theory

recall that the producer’s goal is to maximize profit, \( \Pi = TR - TC \)

so far we’ve examined the cost side of this problem
[draw up the total and average/marginal cost curve diagrams as review]

note that MC cuts AC and AVC at inflection points and that if MC > AC (and same for AVC), then AC is rising

we can now redefine the relevant planning horizons as the case when at least some costs are fixed vs. the case when all costs are variable.

We’re almost ready to solve the producer’s problem, namely what output to make in order to maximize profit, but let’s spend a little time first looking at the revenue side of the problem
recall that TR, or total revenue, is simply price times quantity, \( TR = PQ \)

this can be added to the TC diagram; note that the goal is to have the widest possible vertical difference between TR and TC with \( TR > TC \) in order to maximize profit; note at the crossing points, profit = 0

![Graph showing TR and TC curves]

average revenue \( AR = \frac{TR}{Q} = P \)

and marginal revenue \( MR = \frac{dTR}{dQ} \)

Note that both MR and MC are functions of Q in general, MR(Q) and MC(Q)

now, in order to maximize profit, the producer picks the value of Q where

\[
\frac{d\Pi}{dQ} = 0, \text{ so } \frac{d\Pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ} = MR(Q) - MC(Q) = 0
\]

so optimal Q is where MR = MC

This is the general rule that we assume all firms follow

Now we will concentrate on an important special case of this general point. If firms take price \( P \) as fixed, then that is the same as saying that MR = P, because the amount of additional revenue from selling another unit is exactly equal to P
So in this case, the firm simply sets $P = MC$ and solves this equation for $Q$ (where $P$ is a constant and $MC$ is a function of $Q$)

Let’s try doing this in a numerical example

so to start one of these problems, all I have to give you is the cost function and the value for $P$:

$$C = Q^2 - 4Q + 16 \ ; \ P = 6$$

what are $VC$, $FC$, $AC$, $AVC$, and $AFC$?

$$MC = 2Q - 4$$

set $P = MC$, so $Q^* = 5$

And $\Pi = TR - TC = 6\times5 - (25 - 20 + 16) = 30 - 21 = 9$
one last consideration: the firm should not continue to operate if it loses too much money. It’s going to have to its fixed costs even if it doesn’t operate, so the main concern in the short run is not to lose any more than that by continuing to operate (e.g., still have to pay your lease)

so the short run shutdown condition is that the firm must cover its variable costs

this can be shown as $\text{TR} \geq \text{VC}$, and dividing through by $Q$, this is the same as $\text{P} \geq \text{AVC}$

(the shutdown condition is also equivalent to $\text{II} \geq -\text{FC}$; manipulate it to convince yourself that this is the case)

this allows us to define supply: the firm’s marginal cost curve above average variable cost is the SR/MR supply curve

note that in the long run the firm should not continue to operate if it does not cover all of its costs (or think of that all costs become variable in the LR); i.e. contracts expire and the firm is freed of fixed cost obligations that existed in the short run.

So the long run shutdown condition is that profits must be nonnegative:

$$\text{TR} \geq \text{TC}, \text{ and dividing through by } Q, \text{ this is the same as } \text{P} \geq \text{AC}$$

so the firm’s marginal cost curve above average cost is the LR supply curve (because all the fixed costs become variable in the LR)

this implies that the LR supply curve must be more elastic than the SR supply curve, because it includes more frequently having firms exit from the industry when they aren’t covering their total costs.

What if $\text{P}$ drops below $\text{AVC}$?

Then the firm should stop operating immediately as it is losing more money by continuing to produce than if it just closed (in which case it only loses its fixed costs).
Let’s check the SR and LR shutdown conditions for our numerical example. At the current P, profits are positive, so the firm should continue to operate in both the short run and the long run.

What if P drops to 2?
Then solves to $Q^* = 3$, $TR = 6$, $TC = 9 - 12 + 16 = 13$, so profit = $-7 > -16 = -FC$
Then the firm should continue to operate in the SR, but once their lease is up (or whatever is generating their fixed costs) they should shut down.

Note we can solve for the LR break-even point (and the SR break-even point) by setting $MC = AC$ and solving for the P and Q at this point (in the SR, set $MC = AVC$)
remember this is because MC cuts AC at its minimum (and also AVC if the minimum is in the first quadrant)

$$MC = 2Q - 4 = Q - 4 + \frac{16}{Q} = AC; \text{ rearrange to get Q on the left: } Q^2 = 16, \text{ so } Q = 4$$
at this point $MC = 2\times4 - 4 = 4$ and since firms set $MC = P$, $P$ must = 4 as well at this Q

(and note that $AC = 4$ as well at this point)

and can confirm that profit = 0: $TR = 4\times4 = 16$; $TC = 16 - 16 + 16 = 16$
I. 1) -1/2 (assuming K is graphed on the vertical axis)

2) K = 20, L = 1

3) 41

II.

1) 

2) 

Answers to Practice Problems from 2/11/11
I. Consider the following cost function:

\[ C = Q^2 - 2Q + 4 \]

1) What is the formula for VC? What is FC?

2) What is the formula for AC? For AVC? For AFC? For MC?

3) At what value of Q is AC minimized? What is the value of AC at this level? What is the value of MC?

4) Sketch AC, AVC, AFC, and MC on the same graph and indicate the point where AC is minimized

II. Assume P = 4

1) What is the firm’s optimal level of output Q*?

2) Add a line for P on your cost curve diagram from I.4) and indicate Q* on the diagram.

3) What is TR? What is TC? What is \( \Pi \)?

4) Confirm that the shutdown condition is met (i.e., that the firm should produce at this level Q* instead of at Q = 0)