

12th Class

2/16/11

go over practice problems

go over handout/overhead summarizing the firm's problem

now we segue from producer theory back into markets, coming in from the supply side

so far we've been showing applications of the first main mathematical principle underlying economic theory: optimization; now we will focus on the other main mathematical principle: equilibrium

first, let's sum back up to market demand and market supply

we did market demand already; let's do a similar exercise of summing up individual firm supply curves to get the market supply curve

note for the firms we start with the cost function, get the marginal cost curve, and that is the firm's supply curve (subject to the shutdown condition)

consider the firm's cost function that we were working with last class:

$$C = q^2 - 4q + 16$$

note I'm going to switch to using a little q for cases when I am talking about a single firm (or consumer) and save big Q for when I am talking about market supply or demand

$$\text{then the firm's marginal cost } MC = 2q - 4$$

and if the firm is a price-taker, the firm solves for its optimal q by setting $MC = P$:

$$P = MC = 2q - 4$$

$$2q = P + 4, \text{ so } q = .5P + 2$$

so this is the firm's supply curve (subject to caveat that it would only operate above AVC in the SR and above AC in the LR, so would still need to check the shutdown condition to make sure that it isn't better to produce at $q = 0$)

Now suppose you had multiple firms in the industry, all with the same supply curve; then just sum up all of their individual quantities to get the market supply:

$$Q_S = \sum_n q = \sum_n (.5P + 2)$$

for instance, if there are 100 firms, $Q_S = 100(.5P + 2) = 50P + 200$

This is exactly parallel to how we summed individual demand curves to get market demand.

Assume for simplicity that a bunch of consumers have the same demand curve:

$$q_D = 16 - 1.25P$$

$$\text{then } Q_D = \sum_n q_D = \sum_n (16 - 1.25P)$$

for instance, if there are 200 consumers, $Q_D = 200(16 - 1.25P) = 3200 - 250P$

And now that we have a market supply curve and a market demand curve, we can solve for equilibrium:

$$50P + 200 = 3200 - 250P; 300P = 3000, P = 10, \text{ so } Q = 700$$

since the firms and consumers are identical, we just divide the total output by their numbers to find out how much each firm sells (7) and how much each consumer buys (3.5)

and we can check the individual firm's profit to see whether they satisfy the shut-down conditions in the SR and the LR:

$$\Pi = 10 \cdot 7 - (49 - 28 + 16) = 70 - 37 = 33, \text{ so continue to operate}$$

it's interesting to consider the dynamics of firm entry and exit. Note that we have argued that if there is insufficient profit then firms will exit the industry. What if there is excess profit (anything greater than zero)?

We need to make clearer the assumptions governing this market. We have already assumed that individual firms (and consumers) are price-takers, in other words they look at the price and decide what to do based on it. They do not see themselves as influencing the price by their individual actions. Yet of course they are influencing the price by their joint decisions to sell and buy.

And thus, when firms leave the market, the supply curve shifts to the left, price goes up, and quantity supplied goes down. This mechanism continues until enough firms have left that the remaining ones can cover their costs at the new higher price.

We don't specify clearly which firms leave and which stay, but note they have to leave rather than simply cutting back on output because only then do we save their fixed costs

Secondly, if profits rise, we posit the opposite mechanism: new firms enter. Then the supply curve shifts to the right. This assumes free entry just as we have assumed free ability to exit the market (i.e., firms aren't forced to stay in markets when they are losing money, and firms can enter markets at will if they so choose to make the investment). If this happens, the price falls and profits fall to zero

All this is assuming the same existing technology. But other things can also shift the supply curve for individual firms and thus the market, or industry, supply curve. These other things can include changes in the factor prices and changes in technology. A drop in one or more factor prices, and an improvement in technology, leads to a downward shift in each firm's cost curves and an increase in the market supply curve, leading to a drop in price.

[discussion of genetically modified salmon; grow twice as fast, so cost curve/supply curve shifts downwards

<http://www.npr.org/templates/story/story.php?storyId=129984367>]

Firms who fail to take advantage of these innovations or to change their factor mix will be unable to cover their higher costs when the price falls in the market, and will be driven out.

And thus we see that several things must be true in LR equilibrium in a perfectly competitive market:

- we would see no firms or consumers exiting or entering the market (hence it is an equilibrium)
- all firms have the same cost conditions (i.e., the same marginal and average costs)
- all firms are making exactly zero profit (i.e., exactly but no more than covering their costs-- note that costs include all sorts of opportunity costs. For instance owner-operators have to decide that operating their firm is just as good as their next-best option.)

note we will later see cases where there is an equilibrium in the sense that no firm exits or enters the market, but the firm or firms in the market will be making positive profits. Thus this must be a case where there is some sort of barrier to entry into the market so that potential entrants cannot enter.

Next class I will make the argument that intervention in a perfectly competitive market will have implications for economic efficiency and distribution. First we'll need to define efficiency and distribution, and show how losses in efficiency as well as redistributive effects of interventions can be measured.

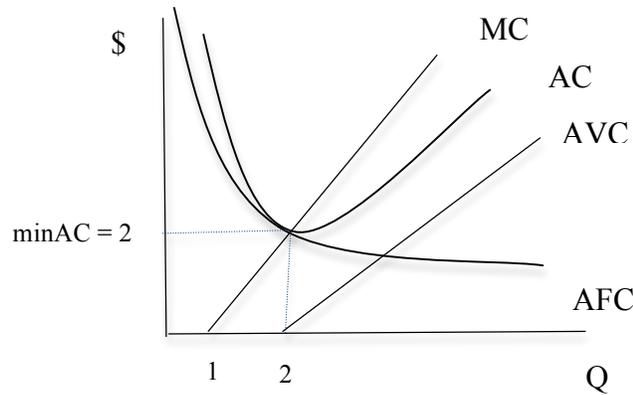
Answers to Practice Problems from 2/14/11

I. 1) $VC = Q^2 - 2Q$; $FC = 4$

2) $AC = Q - 2 + \frac{4}{Q}$; $AVC = Q - 2$; $AFC = \frac{4}{Q}$; $MC = 2Q - 2$

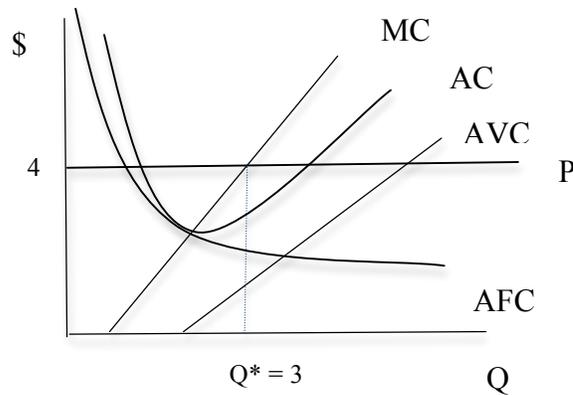
3) at $Q = 2$, $AC = 2$, $MC = 2$

4)



II. 1) $Q^* = 3$

2)



3) $TR = 12$; $TC = 7$; $\Pi = 5$

4) $5 > -4$, so produce at $Q^* = 3$

Handout on the Firm's Problem 2/16/11

Consider the production function $Q = Q(K, L)$

and the cost of production: $TC = rK + wL + FC$

and assume the firm takes input/factor prices (r, w) , fixed costs (FC) , and output price P as given.

Then for a given level of output, say Q^* , there are 2 associated factor demand functions:

$$K = K(r, w, Q^*)$$

$$L = L(r, w, Q^*)$$

The firm chooses the optimal level of output Q^* that maximizes profit, where profit is:

$$\begin{aligned}\Pi &= TR - TC \\ &= PQ - rK - wL - FC \\ &= PQ(K, L) - rK - wL - FC\end{aligned}$$

The cost-minimizing choices of K and L for producing this level of output occur where the marginal rate of technical substitution equals the factor price ratio:

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}, \text{ or } \frac{MP_L}{w} = \frac{MP_K}{r}$$

Assuming the firm has correctly cost-minimized, we can write the minimum cost of achieving a given level of output as $TC = C(Q)$, and then we can write the profit function as:

$$\Pi = PQ - C(Q)$$

To maximize Π by choosing the optimal level of output, set:

$$\frac{d\Pi}{dQ} = 0, \text{ so } P - \frac{dC}{dQ} = 0, \text{ or } P - MC(Q) = 0, \text{ or } P = MC$$

And in general you can solve this single equation for the potentially optimal level of output Q^*

Then check the shut-down condition: so long as $\Pi(Q^*) \geq -FC$ produce Q^* ; otherwise set $Q = 0$

This framework can be generated to any number of inputs/factors (represented by 1, 2, ..., n):

Consider the production function $Q = Q(X_1, X_2, \dots, X_n)$

and the cost of production: $TC = P_1X_1 + P_2X_2 + \dots + P_nX_n + FC$

Then for a given level of output Q^* there are n associated factor demand functions:

$$X_1 = X_1(P_1, P_2, \dots, P_n, Q^*)$$

$$X_2 = X_2(P_1, P_2, \dots, P_n, Q^*)$$

...

$$X_n = X_n(P_1, P_2, \dots, P_n, Q^*)$$

The firm chooses the optimal level of output Q^* that maximizes profit, where profit is:

$$\Pi = TR - TC$$

$$= PQ - P_1X_1 - P_2X_2 - \dots - P_nX_n - FC$$

$$= PQ(X_1, X_2, \dots, X_n) - P_1X_1 - P_2X_2 - \dots - P_nX_n - FC$$

And the cost-minimizing choices of inputs occur where:

$$\frac{MP_1}{P_1} = \frac{MP_2}{P_2} = \dots = \frac{MP_n}{P_n}$$

And the rest of the firm's problem continues as in the two-factor case once $C(Q)$ is calculated.

Practice Problems 2/16/11

I. Assume there are 50 identical consumers, each with the demand curve:

$$q = 7 - P$$

What is the market demand curve?

II. Assume there are 50 identical firms, each with the cost function:

$$C = q^2 - 2q + 4$$

1) What is each firm's supply curve?

2) What is the market supply curve?

III. Consider the market described in **I.** and **II.**

1) What are the equilibrium price P and quantity Q ?

2) Calculate profits for a firm. Will there be any exit or entry occurring in this market?