

14th Class

2/21/11

do practice problem answers

Now we will relax the assumption that firms are price takers and consider what happens if they face a downward-sloping demand curve for their product.

Let's first review the relationship between TR, P, MR, and the elasticity of demand

recall  $TR = PQ$ , so  $\frac{dTR}{dQ} = P + Q \frac{dP}{dQ}$ , since P is an implicit function of Q:

$$P = P(Q), \text{ e.g. } P = 100 - 2Q$$

$$\text{so } MR = \frac{dTR}{dQ} \text{ by definition, and } MR = P \left[ 1 + \frac{Q}{P} \frac{dP}{dQ} \right]$$

$$\text{and } MR = P \left[ 1 + \frac{1}{\epsilon} \right] = P \left[ 1 - \frac{1}{|\epsilon|} \right]$$

note if elasticity=1, then  $MR = 0$ ;

if elasticity > 1 (in absolute value), then  $0 < MR \leq P$ ; in the limit,

as elasticity goes to infinity,  $MR = P$

if elasticity < 1, then  $MR < 0$  (<P)

so MR always lies on or below price.

$$\text{also } \frac{dTR}{dP} = Q + P \frac{dQ}{dP} = Q [ 1 + \epsilon ] = Q [ 1 - |\epsilon| ]$$

so if elasticity=1,  $\frac{dTR}{dP} = 0$ ; so unit elasticity means changing price does not change revenue;

if elasticity > 1 (in absolute value),  $\frac{dTR}{dP} < 0$ , so elastic demand means raising the price reduces total revenue;

and if elasticity < 1,  $\frac{dTR}{dP} > 0$ , so inelastic demand means raising the price increases total revenue.

Now notice that for a linear demand curve  $Q = a - bP$

$$\frac{dQ}{dP} = -b, \text{ so } \epsilon = \frac{P}{Q} \frac{dQ}{dP} = \frac{bP}{a-bP}$$

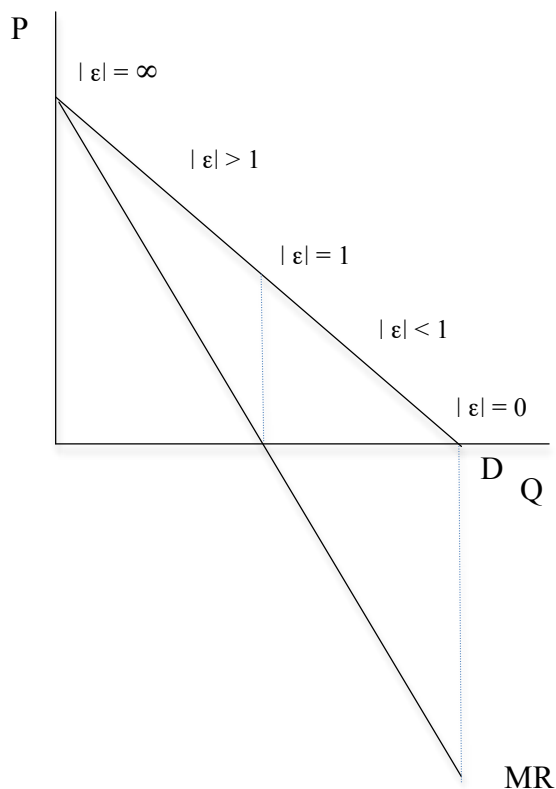
using the inverse demand curve,  $P = \frac{a}{b} - \frac{1}{b} Q = a' - b'Q$

$$\text{therefore } MR = \frac{dTR}{dQ} = P + \frac{dP}{dQ} Q = P - b'Q = a' - b'Q - b'Q$$

so the formula for MR for a linear demand curve is:

$$MR = a' - 2b'Q$$

so the marginal revenue curve has twice the slope of the inverse demand curve:  $P = a' - b'Q$ .



consider for instance the case in which a firm faces the demand curve for the entire market  
 This is known as a monopoly and is characterized as being one seller facing many buyers  
 so the firm is no longer a price-taker, but consumers still are

The monopolist's problem is, as for any firm, to maximize profit

$P = TR(Q) - TC(Q)$  by choosing  $Q$ , where  $TR(Q) = P(Q)Q$

so  $\frac{dTR}{dQ} - \frac{dTC}{dQ} = 0$ , or  $\frac{dTR}{dQ} = \frac{dTC}{dQ}$ , or  $MR = MC$ , is the profit-maximizing condition.

remember  $MR = \frac{dTR}{dQ} = P(Q) + Q \frac{dP}{dQ} = P(Q) \left[ 1 - \frac{1}{|\epsilon(Q)|} \right]$

if  $|\epsilon| < 1$ ,  $MR$  is negative,

so  $|\epsilon| \geq 1$  is where the monopolist produces.

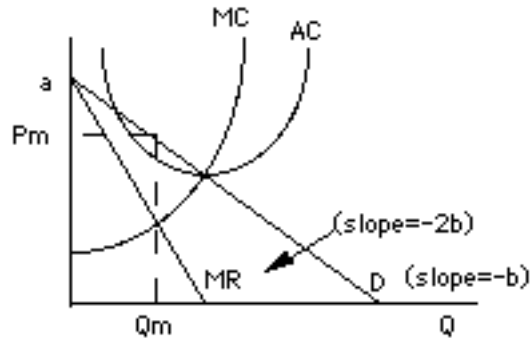
Note that a monopolist has no supply curve; as demand shifts around it will not trace out a supply curve, because the elasticity of demand always matters in determining quantity produced.

Here is the linear case:

the inverse demand curve is  $P(Q) = a' - b'Q$

$TR(Q) = a'Q - b'Q^2$ , so  $MR = \frac{dTR}{dQ} = a' - 2b'Q$

so  $MR$  has twice the slope of the demand curve:



(note I redefined  $a'$  as  $a$  and  $b'$  as  $b$  in the picture above as it is an older picture I had made)  
 set  $a' - 2b'Q = MC$  and solve for  $Q^*$ , then plug  $Q^*$  into  $P(Q)$  to find the price the monopolist charges. Then  $\Pi = P^*Q - C(Q)$ .

Example: Market demand is  $Q_d = 1000 - 10P$  and the monopolist has the cost function  $C(Q) = 100 + 10Q + .05Q^2$ .

to solve for equilibrium quantity, set  $MR = MC$

$$P = 100 - .1Q, \text{ so } MR = 100 - .2Q; MC = 10 + .1Q$$

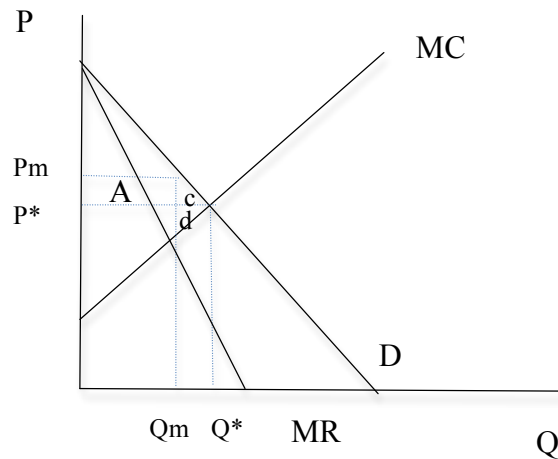
$$100 - .2Q = 10 + .1Q; .3Q = 90, Q = 300$$

$$\text{so } P = 100 - .1(300) = 70$$

$$\text{and } \Pi = (70)(300) - [100 + 10(300) + .05(300)^2]$$

$$= 21,000 - 7600 = 13,400.$$

We can show that a monopoly is Pareto-inefficient:



a monopolist causes DWL (c+d) relative to the solution where demand = MC (as well as a transfer from consumers to producers of A) and both the consumers and producer could be made better off if more units were sold at a lower price.

In other words, I can come up with a situation that both the consumers and the producer would prefer (namely price discrimination on the extra units of output).

When might we want a monopoly?

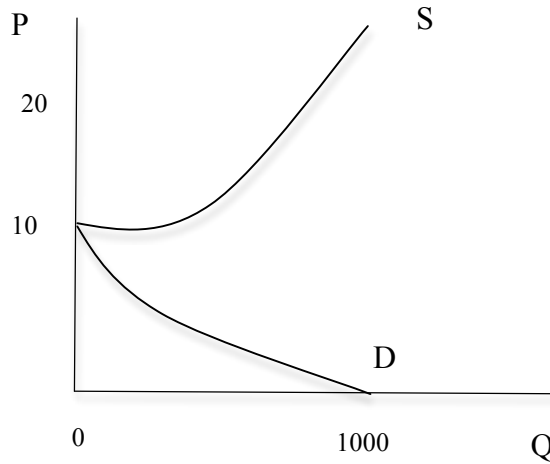
- a patent as a limited monopoly to encourage R&D
- a natural monopoly: demand not large enough to sustain more than one firm

Causes of monopoly:

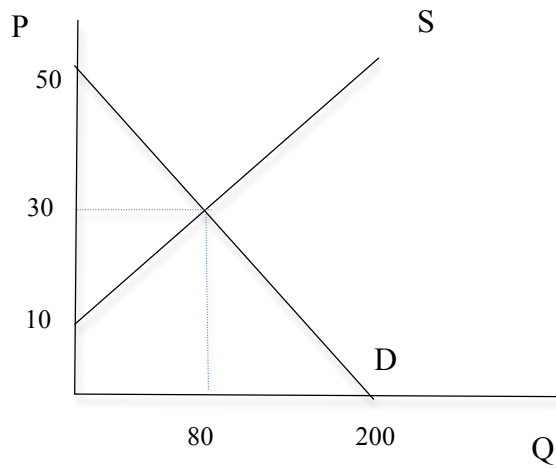
- minimum efficient scale very large (natural monopoly)
- cartel/collusion
- incumbent cost advantage

Answers to Practice Problems from 2/18/11

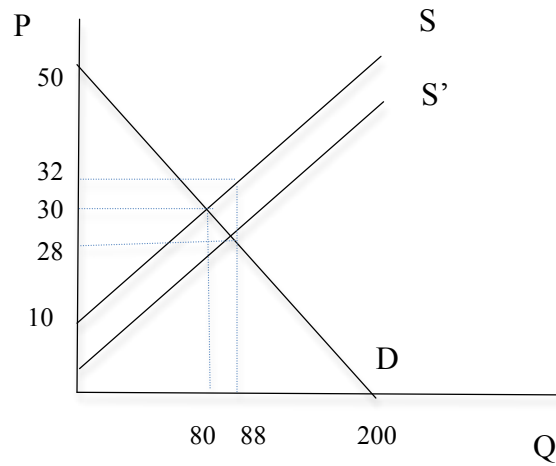
I. 1)  $Q = 0, P = 20$  (no goods traded at that set price)



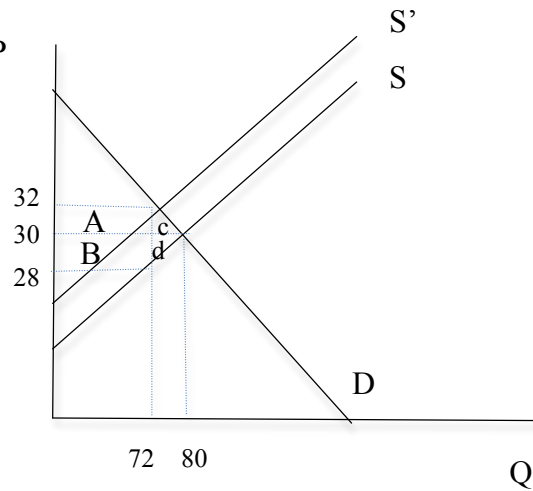
2)  $Q = 80, P = 30$



I. 3)  $Q = 88, P_S = 32, P_D = 28$



II. 1)



2) change in CS =  $A+c = 2*72 + .5*2*8 = 152$ ; change in PS =  $B+d = 152$

3) tax revenue =  $A+B = 4*72 = 288$

4) DWL =  $c+d = .5*4*8 = 16$

Practice Problems 2/21/11

I. A monopolist with cost function  $C = \frac{1}{8}Q^2 + 10Q + 800$   
faces the market demand curve  $Q = 200 - 4P$

- 1) What is the inverse demand curve? What is the marginal revenue curve?
- 2) What is the marginal cost curve?
- 3) What is the monopolist's optimal quantity and price?

II. Consider the same cost and demand conditions as in I.

- 1) If the monopolist were forced to set price equal to marginal cost (so forced to act as if they were in perfect competition) what would be the firm's quantity and price?
- 2) Sketch a graph illustrating the inverse demand, marginal revenue, and marginal cost curves for the firm and situation under monopoly and under "perfect competition."
- 3) Mark on the graph the deadweight loss caused by the monopolist being allowed to do what it wants as opposed to being forced to act as if it were in perfect competition.