let’s continue with our simple model and add the third sector to our economy: the government

so now \( AD = C + I + G \)

we don’t assume to know why the government does what it does (black box) so we just set

\( G = G_0 \)

this will increase \( AD \) in our simple model by that amount times the multiplier:

re-solving our system so far:

\[
Y = C + I
\]

\[
C = C_0 + C_1 Y
\]

\[
I = I_0
\]

\[
G = G_0
\]

substituting into the first equation: \( Y = C_0 + C_1 Y + I_0 + G_0 \)

so \( Y(1 - C_1) = C_0 + I_0 + G_0 \)

and \( Y = \frac{1}{1-C_1}(C_0 + I_0 + G_0) \)

hence we see that government spending has a stimulatory effect on the economy
continue our simple numerical example:

starting point (from last class):

\[ C = 10 + .80Y, \quad I = 50 \]

so \[ Y = \frac{1}{1 - .8} (10 + 50) = \frac{1}{.2} (60) = 5 \times 60 = 300; \quad C = 250 \]

now:

\[ C = 10 + .80Y, \quad I = 50, \quad G = 30 \]

so \[ Y = \frac{1}{1 - .8} (10 + 50 + 30) = \frac{1}{.2} (90) = 5 \times 90 = 450; \quad C = 370 \]

but you might well ask how government is supposed to pay for its spending. Here there isn’t any requirement that the government balance its budget. So far it is running a deficit,

Let’s allow for taxes \( T \) as well in the model so as to have a way to pay for government spending:
introduce a wedge between national income and disposable (or personal) income. Disposable income is what the consumers have net of taxes, so change the consumption function:

\[ Y_d = Y - T \]

\[ C = C_0 + C_1 Y = C_0 + C_1 (Y - T) \]

we’ll try out taxes of two types, a fixed amount and tax as a percent of income (an income tax)

\[ T = T_0 \]

solve for \( Y \) now:

\[ Y = C_0 + C_1 (Y - T_0) + I_0 + G_0 \]

\[ Y = \frac{1}{1 - C_1} (C_0 - C_1 T_0 + I_0 + G_0) \]

So spending is reduced by \( -\frac{C_1}{1 - C_1} \) times \( T_0 \)

so the larger the marginal propensity to consume, the larger the reduction caused by the tax

e.g. \( C = 10 + .80 Y, I = 50, G = 30, T = 20 \)

so \( Y = 10 + .80 (Y - 20) + 50 + 30 \)

so \( Y = \frac{1}{1 - .8} (10 - 16 + 50 + 30) = \frac{1}{2} \cdot 74 = 370; \ C = 290 \)

define the government budget as \( T - G \) (deficit if negative, surplus if positive)

What if we forced the government to balance its budget each period, so that \( G = T \)? Would this have any effect on the economy?
Yes! Let’s show this:

\[ Y = C_0 + C_1 (Y - G_0) + I_0 + G_0 \]

\[ Y = \frac{1}{1-C_1} (C_0 + I_0 + (1-C_1)G_0) = \frac{1}{1-C_1} (C_0 + I_0) + \frac{1-C_1}{1-C_1} G_0 = \frac{1}{1-C_1} (C_0 + I_0) + G_0 \]

The first round of spending pushes out AD, even though future rounds are halted by the tax being pulled out of income, so the balanced budget multiplier is 1!

e.g. \( C = 10 + .80Yd, I = 50, G = 30, T = 30 \)

so \( Y = 10 + .80(Y - 30) + 50 + 30 \)

so \( Y = \frac{1}{1-.8} (10 - 24 + 50 + 30) = \frac{1}{.2} (66) = 5*66 = 330; \ C = 250 \) (compare to starting point)

Now let’s allow for an income tax instead:

\[ T = T_1 Y, \ 0 \leq T_1 \leq 1 \]

now this means that government shares in good times and also shares the burden (by taking in lower tax revenues) in bad times; this is an example of an automatic stabilizer

note that this will now affect the multiplier unlike the fixed tax

let’s re-solve for Y:

\[ Y = C_0 + C_1 (Y - T_1 Y) + I_0 + G_0 \]

\[ Y(1 - C_1 (1 - T_1)) = C_0 + I_0 + G_0 \]

\[ Y = \frac{1}{1-C_1(1-T_1)} (C_0 + I_0 + G_0) = \frac{1}{1-C_1 + C_1T_1} (C_0 + I_0 + G_0) \]
since we’re adding another positive term in the denominator, the tax rate has the effect of reducing the multiplier effect (making the AD line flatter)
[note extreme case: if the tax rate were 100% then there would be no multiplier at all]

e.g. $C = 10 + .80Yd, I = 50, G = 30, T = .25Y$

so $Y = 10 + .80(Y - .25Y) + 50 + 30$

so $Y = \frac{1}{1 - .8(1 -.25)} (10 + 50 + 30) = \frac{1}{.4} (90) = 2.5*90 = 225; \ C = 145$

so government spending is expansionary and taxation is contractionary

more general case can allow for both a fixed and variable tax: $T = T_0 + T_1Y$

Since we don’t in fact make the federal government balance its budget (though many state and local governments do have balanced budget rules), we can run a federal deficit year after year here’s the Congressional Budget Office’s most recent estimate on the federal govt deficit:

So how do we pay for the deficit in any given year?
By issuing government bonds, i.e. borrowing from individuals (including individuals outside the US), and agreeing to pay back the money with interest in the future (where interest payments then become another part of government spending)

in a more sophisticated formal model than we are able to develop in this class, we would include bonds fully in the model and show how they affect AD as well; I’ll allude to some possible effects as we go along later in the class

If we run a deficit year after year, we accumulate debt (even as we pay off some of our outstanding bonds from year to year as well)

you can admire the current level of the national debt at the national debt clock:
http://www.brillig.com/debt_clock/

note some ways this simple model could be extended:
--allow for more than one type of consumer in the economy, where they could have different marginal propensities to consume and different tax rates
--allow for some portion of taxes to be transfers between types of consumers, and thus affect the overall multiplier by moving say from low-spending consumers to high-spending consumers
Answers to Practice Problems from 3/21/11

I. 1) Y = 600; C = 550

\[ 2) \text{ multiplier} = \frac{1}{1 - .75} = \frac{1}{.25} = 4 \]

3)

II. 1) Y = 640; C = 580

2) see diagram
I. An economy can be characterized by the following four equations:

Consumption function: \( C = 100 + .75Y \)

Investment: \( I = 50 \)

Government: \( G = 100 \)

Equilibrium: \( Y = C + I + G \)

1) Solve for the equilibrium values of \( Y \) and \( C \).

2) What is the multiplier?

II. Now suppose the government imposes a lump-sum tax to pay for its spending:

\( T = 100 \)

and the consumption function changes to \( C = 100 + .75Y_d \), where \( Y_d = Y - T \)

1) Now what are the equilibrium values of \( Y \) and \( C \)?

2) What is the multiplier?

III. Suppose that instead of a lump-sum tax, the government imposes an income tax:

\( T = .2Y \)

1) Now what are the equilibrium values of \( Y \) and \( C \)?

2) What is the multiplier?