up until now we’ve assumed that investment is basically some unfathomable decision by firms

this is in keeping with Keynes’s view that investment is determined by “animal spirits” and thus essentially unpredictable

but today we’ll allow for two possible influences on investment, interest rates (as representing the opportunity cost of investing in physical capital, namely the return on financial capital) and current output (an accelerator model of the economy)

what determines the interest rate?

[draw the money diagram and explicate it]
\[ M_D = M_0 - M_1r + M_2Y, \quad M_S = \bar{M}, \quad M_D = M_S \]
shifts can occur through changes in the desire for money and through transactions demand

[discuss term structure of interest rates and risk premia to allow for a multiplicity of interest rates]

why would interest rates affect investment?

we have developed present value calculations as a way to compare investments of different lengths and payback structures

imagine your investment will pay off with a stream of payments over n years in amount \( P_i \) per year, and that the interest rate is \( r \)

note you should be indifferent between receiving \( P_1 \) a year from now and \( P_1/(1+r) \) now

just as you are indifferent between receiving \( P_0 \) now and \( P_0(1+r) \) a year from now

e.g a dollar now or $1.03 a year from now if the interest rate is 3%, because you could put the dollar in the bank and collect $1.03 a year from now.

so present value of an investment

\[
PV = P_0 + \frac{P_1}{1+r} + \frac{P_2}{(1+r)^2} + \ldots + \frac{P_n}{(1+r)^n} = \sum_{i=0}^{n} \frac{P_i}{(1+r)^i}
\]
(if you want continuous compounding and cash flow can state this as an integral instead)

note that payments can be negative; in particular you might invest some amount up front (a cost) in order to get a stream of benefits in the future

perpetuity case: constant payment $P$ per year forever: $PV = P/r$
this makes very clear that as $r$ rises, the value of a perpetuity falls
a perpetuity is one form of a bond, so in general note that as $r$ rises, bond values fall (i.e., you should be willing to pay for a bond today the price exactly equal to its present value)

note that yield on a bond is the payment $P$ divided by the price you actually have to pay so yield = $P/\text{Price}$

one can also calculate the internal rate of return for an investment
this is the interest rate (as used for $r$ in the above formula) for which the PV of the investment is exactly zero. This accounts for the cost of the investment as well as the benefits of the investment

(redo the PV calc showing each year as the difference between benefits $B$ and costs $C$) solve this for the $r$ that makes $PV = 0$

then clearly you should do the higher rate of return investments first (note there is no uncertainty concerning future payments in this model!!)

[draw and motivate investment function]
this orders investments in declining order of payoff as measured by their internal rate of return

write down investment function
show how money supply increase lowers $r$, increases investment and thus $AD$

then show feedback effect through expanded money demand equation—transactions demand leads to increased money demand, which offsets interest rate change through Ms shift

final avenue for effects on investment: accelerator model. Investors think current state of economy is good predictor of future state, so they invest more when economy is high and less when economy is low: $I = I_0 - I_1r + I_2Y$
[note this is just a fist pass at how to model expectations]

so now note that over the course of the last week and a half we have developed a fairly elaborate model of the economy, characterized by the following nine equations:

\[ C = C_0 + C_1 Y_d \]
\[ Y_d = Y - T \]
\[ I = I_0 - I_1r + I_2 Y \]
\[ T = T_0 + T_1 Y \]
\[ G = G_0 \]
\[ M_D = M_S \]
\[ M_D = M_0 - M_1r + M_2 Y \]
\[ M_S = \bar{M} \]

This system can be solved for the equilibrium values of \( Y, C, I, T, r, Y_d, \) and \( M_D \). Everything else is set exogenously (given).

note we can describe the multiplier algebraically as well:

\[
Y = \left( \frac{1}{1 - C_1 + C_1 T_1 + I_1 \frac{M_2}{M_1} - I_2} \right) \left( C_0 - C_1 T_0 + I_0 - I_1 \frac{M_0 - \bar{M}}{M_1} + G_0 \right)
\]

where the multiplier is in the first set of parentheses

so any change in an element in the second set of parentheses can be traced through to the change in \( Y \)

similarly, any change in an element in the multiplier changes the value of the multiplier and thus also changes \( Y \)

the missing thing in this model is: aggregate supply, or in other words, any kind of adjustment in prices

indeed, we don’t have price in this model at all

The task over the next two lectures will be to bring aggregate supply and prices into this model
Answers to Practice Problems from 3/25/11

I. 1) .10, or 10%

2) 10

II. 1) $9,000; $100,000

2) reserves rise by $500; loans rise by $4500; deposits rise by $5000; total money supply rises to $115,000
I. An economy can be characterized by the following seven equations:

- **Consumption function:** \( C = 100 + 0.75Y \)
- **Investment function:** \( I = 50 - 5r \)
- **Government:** \( G = 100 \)
- **Money demand:** \( M_D = 100 - 10r \)
- **Money supply:** \( M_S = 80 \)
- **Money market equilibrium:** \( M_D = M_S \)
- **Goods market equilibrium:** \( Y = C + I + G \)

1) Solve for \( r \). Then what is \( I \)?

2) Solve for the equilibrium value of \( Y \).

3) What is the multiplier?

II. Now suppose the money demand function becomes:

\[ M_D = 100 - 10r + 0.1Y \]

1) Now what are the equilibrium values of \( Y \), \( r \), and \( I \)?
   (hint: solve for \( r \) as a function of \( Y \) from the money market equilibrium and substitute into the goods market equilibrium)

2) What is the multiplier?

III. Now suppose in addition to II. that the investment function becomes:

\[ I = 50 - 5r + 0.1Y \]

1) Now what are the equilibrium values of \( Y \), \( r \), and \( I \)?

2) What is the multiplier?