

6th Class

2/2/11

Review assumptions from last time: people try to maximize their utility; utility is generated by consumption (indirectly by income); more is better, but at a diminishing rate; preferences are stable, complete, and consistent (including transitive); averages are preferred to extremes

Note these assumptions could relate to choice for other animals besides humans; tests for things like transitivity and stable preferences; can draw indifference curves for nonhumans

e.g. experiments with capuchin monkeys

<http://www.plosone.org/article/slideshow.action?uri=info:doi/10.1371/journal.pone.0002414&imageURI=info:doi/10.1371/journal.pone.0002414.g001#>

note the monkeys also understand the abstract concept of money, exchanging tokens for food

now back to idea of non-well-behaved cases. They will generate indifference curves that don't look like the well-behaved one I just drew

do case of red pencils and blue pencils for the colorblind person

note I also assume divisibility in all cases rather than integer constraints

go over problem answers from last time

the case of red pencils and blue pencils is what is called perfect substitutes and there we can write the utility function as $U = R + B$ (in the case on the problem set, can write the underlying utility function as $U = 2D + N$;

the case of left and right shoes for two-footed people is perfect complements and we write $U = \min[L, R]$; the other case is $U = R$ (where one good is a neutral good); the final case is that one good is a "bad" good and we should probably try to reverse the cases (absence of mushrooms rather than presence of them) in order to generate well-behaved indifference curves; e.g. $U = P - M$

mention link that student sent; highlight finding (\$75K key amount), also discusses relative rather than absolute well-being; note we have posited this as absolute, but that's a reasonable alternative formulation (though more difficult in some ways to manipulate mathematically)
<http://www.time.com/time/business/article/0,8599,2016291,00.html>

Very important final caveat: we can't actually observe anyone's utility, but people can tell us if they like one basket of goods better or worse (or the same as) than another

hence there is a distinction between cardinal and ordinal utility
(e.g., horse race finishing order is ordinal; knowing their exact race times is cardinal)

Thus you could actually trace out people's indifference curves if you ask them enough questions, but you don't actually know the numerical labels for the indifference curves

for most purposes in economics, ordinal utility (ordering) will suffice, but there are parts of economics where it is hard to go further without knowing exact utility numbers (uncertainty, welfare economics)

fundamental question of whether it is either possible or valid to make interpersonal utility comparisons (people operate on different feeling scales)

continue with our discussion of consumer theory
today introduce the budget constraint, then solving the consumer choice problem

budget constraint

assume income (I) is given

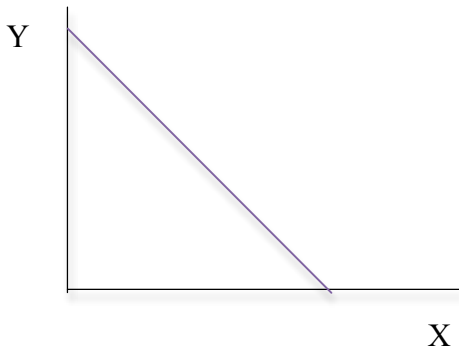
[sidebar on how it is the solution to the labor supply problem; $U=U(\text{consumption}, \text{leisure})$]

the other things the consumer takes as given are the prices of goods and services (e.g., P_X, P_Y)
(note I use X for what is really Q_X and Y for Q_Y ; see it both ways in the econ literature)

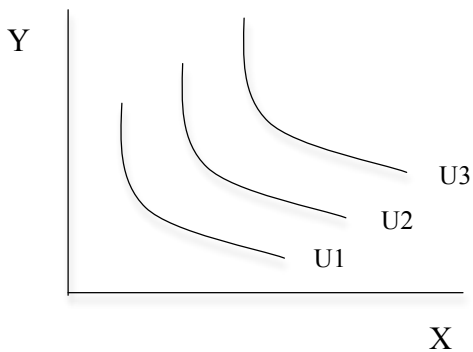
[doesn't rule out negotiating and volume discounts; just assume the whole structure of prices and what is the minimum one might pay for something is all taken into account]

then can represent the consumer's budget constraint as $P_X X + P_Y Y = I$

can graph the budget constraint (show graph) and manipulate it (shifts out or in as income rises or falls; rotates from one point or the other as one or the other price changes); think about the attainable and unattainable points (analogy to PPF)



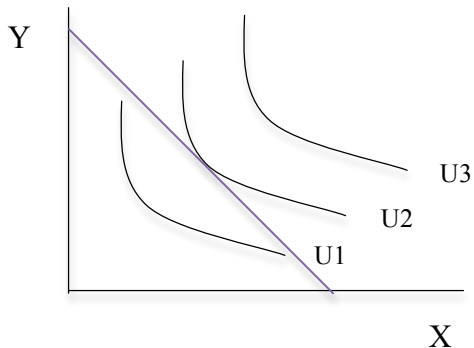
noting that the axes are labelled in the same way, combine it with the indifference curve graph



now consider what the solution point is to the consumer's problem (i.e., trying to reach the highest possible indifference curve which corresponds to the highest possible level of utility, given that one must stay on (or below) the budget constraint).

analogy with mountain climbing subject to a fence [draw picture]

at the optimal point, the slope of the indifference curve will be equal to the slope of the budget constraint (motivate on the graph and with words)



what is the slope of the budget constraint?

rewrite the budget constraint so that Y is a function of X:

$$Y = \frac{I - P_X X}{P_Y} = \frac{I}{P_Y} - \frac{P_X}{P_Y} X, \text{ so } \frac{dY}{dX} = -\frac{P_X}{P_Y} < 0$$

what is the formula for the slope of an indifference curve (also measured as $\frac{dY}{dX}$)?

this can be derived similarly to how we derived the slope of the PPF; utility is obtained from X and Y and a decline in Y generates a loss in utility due to consuming less Y that has to be offset from a gain in utility by consuming more X in order to keep utility constant along the indifference curve:

$$\Delta U = dU = \frac{dU}{dX} dX + \frac{dU}{dY} dY = 0$$

$$\text{so } \frac{dU}{dY} dY = -\frac{dU}{dX} dX \text{ and } \frac{dY}{dX} = -\frac{dU/dX}{dU/dY} = -\frac{MU_X}{MU_Y}$$

we define the marginal rate of substitution, or $MRS = MRS = -\frac{dY}{dX} = \frac{MU_X}{MU_Y}$

so it is equivalent to say that the slope of the indifference curve must be equal to the slope of the budget constraint at the optimal point for the consumer, but (dropping the negative signs) we say that the marginal rate of substitution has to equal the price ratio:

$$MRS = \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

you may prefer for aiding your intuition (and extending it to the case of more than two goods) to rearrange this statement in the following way:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

and note it can then be extended for more goods as a general optimality condition:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y} = \frac{MU_Z}{P_Z} = \dots$$

in other words, for the consumer to be making the optimal choice, the ratio of marginal utility to price has to be the same across all goods consumed; in other words, you want to get the same utility bang for each dollar spent so reallocate your income to increase your total utility.

Thus this means that the consumer is solving a two equation in two unknown (X and Y) system, where one equation is the one we just derived and the other is the budget constraint.

$$1) \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}$$

$$2) P_X X + P_Y Y = I$$

because the marginal utilities are in general functions of X and Y (i.e., your marginal increase in utility depends on how much you're already consuming), usually X and Y appear in the first equation as well. Meanwhile the consumer takes the prices and income as given.

A numerical example of solving for the equilibrium quantities X and Y:

$$U = \ln X + 4 \ln Y ; I = 60; P_X = 2; P_Y = 4$$

so the two equations are:

$$MRS = \frac{dU/dX}{dU/dY} = \frac{1/X}{4/Y} = \frac{Y}{4X} = \frac{1}{2} = \frac{2}{4} = \frac{P_X}{P_Y}, \text{ so } Y = 2X$$

$$\text{and } 2X + 4Y = 60$$

So solve for X and Y by substitution: $2X + 8X = 60$, $X = 6$, $Y = 12$

and can check by plugging these back into the two equations for equality

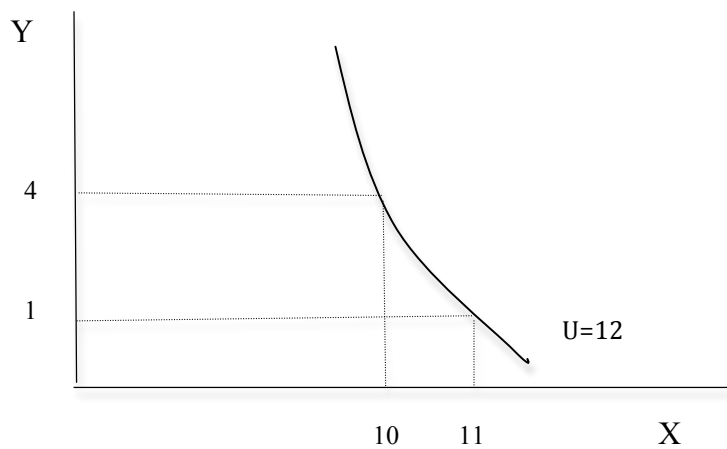
Answers to Practice Problems from 1/31/11

I. 1) $MU_X = 1$; $MU_Y = \frac{1}{2\sqrt{Y}} = \frac{1}{2}(Y)^{-\frac{1}{2}}$

2) $U = 2 + 2 = 4$; $MU_X = 1$; $MU_Y = \frac{1}{4}$

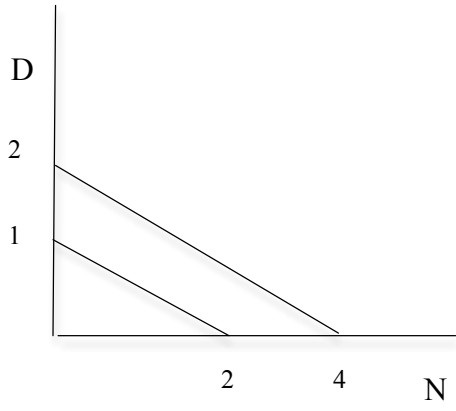
3) There is no diminishing marginal utility with respect to consumption of X.

4) e.g., $X=10$ and $Y = 4$; $X = 11$ and $Y = 1$

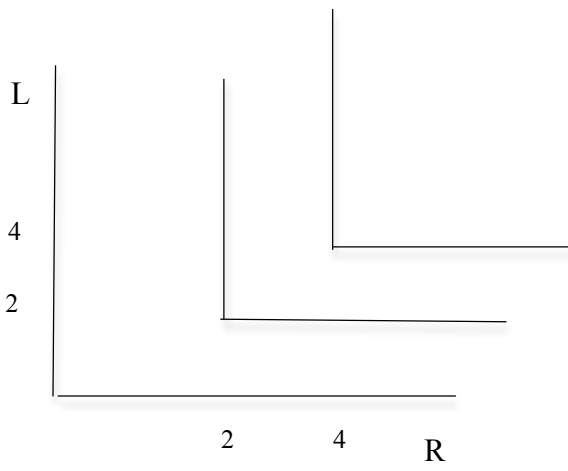


II.

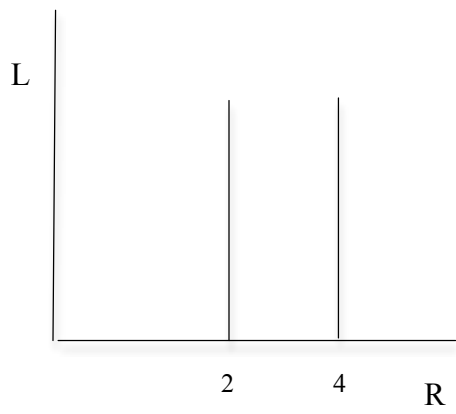
1)



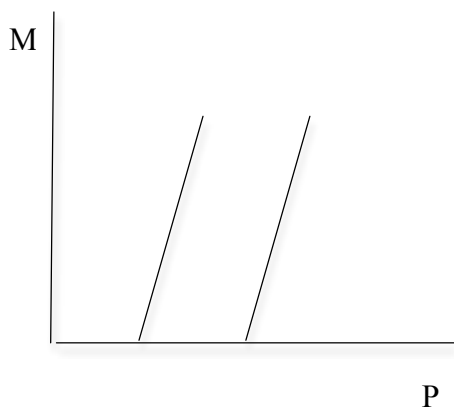
2)



3)



4)



Practice Problems 2/2/11

I. Consider the following utility function (again):

$$U = X + \sqrt{Y}$$

- 1) What is the formula for MRS? Is there anything unusual about this formula?
- 2) If $P_X = 2$, $P_Y = 1$, and $I = 41$, what is the equation for the budget constraint? What is the slope of the budget constraint?
- 3) Solve for the consumer's optimal point on the budget constraint.

II. In each of the following cases, sketch a couple of indifference curves and a budget constraint on the same diagram and indicate where the optimal point is for the consumer.

- 1) Preferences regarding red and blue pencils for a colorblind person; red pencils cost 20 cents each and blue pencils cost 40 cents each.
- 2) Preferences regarding left shoes and right shoes for a person with two feet; right and left shoes cost the same.
- 3) Preferences regarding mushroom and pepperoni as pizza toppings for a person who doesn't like mushrooms but likes pepperoni; the toppings cost the same.