

7th Class

2/4/11

go over problem answers from last time

continue with our discussion of consumer theory

recall the consumer's solution of their optimization problem from last time

today we'll show how the demand function comes from the consumer's solving their optimization problem in response to changes in prices and income

start with income changes; change income up and down and see how the choice varies

We can then plot these changes in a new graph with income on one axis and quantity demanded of one (or the other good) on the other axis; we'll stick with just X for simplicity

note we could then calculate the slope of this function as well.

Classify goods in two large categories: normal (slope positive; actually non-negative) and inferior (slope negative); note a good could be both over different ranges of income

additional idea: how does the percentage change in quantity alter with the change in price?

Introduce important new concept of elasticity

we'll be calculating the elasticity for lots of things; the specific formulae are all special cases of the general idea %change in one thing (almost always Quantity) caused by a %change in something else; if $Y = f(X)$ then the elasticity of Y with respect to X is

$$\varepsilon = \frac{\% \Delta Y}{\% \Delta X} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} = \frac{\frac{dY}{Y}}{\frac{dX}{X}} = \frac{dY}{dX} \frac{X}{Y} (= \frac{d \ln Y}{d \ln X})$$

in the case of the elasticity of quantity demanded with respect to income, we calculate:

$$\varepsilon_I = \frac{\% \Delta Q}{\% \Delta I} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta I}{I}} = \frac{\frac{dQ}{Q}}{\frac{dI}{I}} = \frac{dQ}{dI} \frac{I}{Q} (= \frac{d \ln Q}{d \ln I})$$

so note that once you know the slope of the above curve (called an Engel curve after the German statistician Ernst Engel--not Marx's friend, who noted these patterns in demographic data)

this allows us to make additional distinctions regarding types of goods based on the income elasticity of demand:

$\varepsilon_I = \infty$ is infinitely elastic (perfectly elastic)

luxuries: $\varepsilon_I > 1$

$\varepsilon_I = 1$ is unit elastic

necessities: $0 < \varepsilon_I < 1$

$\varepsilon_I = 0$ is perfectly inelastic

these are all types of normal goods whenever $\varepsilon_I \geq 0$

and inferior goods are whenever $\varepsilon_I < 0$

e.g.: http://en.wikipedia.org/wiki/Income_elasticity_of_demand

- Automobiles 2.46
- Books 1.44
- Restaurant Meals 1.40
- Tobacco 0.64
- Margarine -0.20
- Public Transportation -0.36

We could do a similar exercise varying the price of X. Note this gives us the standard demand curve in P-Q space

and thus calculate both the slope of the demand curve and the elasticity of the demand curve:

$$\varepsilon_{XX} = \frac{\% \Delta X}{\% \Delta P_X} = \frac{\frac{\Delta X}{X}}{\frac{\Delta P_X}{P_X}} = \frac{\frac{dX}{X}}{\frac{dP_X}{P_X}} = \frac{dX}{dP_X} \frac{P_X}{X}$$

this is known as the elasticity of demand, or more technically (for a reason we'll see in a minute), the own price elasticity of demand (for X)

$\epsilon_{XX} = -\infty$ is infinitely elastic demand (perfectly elastic)

elastic demand: $-\infty < \epsilon_{XX} < -1$

$\epsilon_{XX} = -1$ is unit elastic demand

inelastic demand: $-1 < \epsilon_{XX} < 0$

$\epsilon_{XX} = 0$ is perfectly inelastic

these are all types of ordinary goods whenever $\epsilon_{XX} \leq 0$

and Giffen goods are whenever $\epsilon_{XX} > 0$

Note it is very common for sources to drop the sign and report basically $|\epsilon|$ instead of ϵ for own price elasticities of demand, because they are assumed to be almost always nonpositive (i.e., demand curves are assumed to be very rarely upward-sloping.

[interesting question of whether Giffen goods actually occur, even though they are theoretically possible]

[show some examples of calculated elasticities for different products; discuss how broader groupings tend to have lower elasticity]

http://en.wikipedia.org/wiki/Price_elasticity_of_demand

(the wikipedia page preserves the negative sign in its examples)

Note we could also do a similar exercise varying the price of Y and seeing how it affects the quantity demanded of X [show a price rise and a price fall]. We don't usually graph this, but instead just calculate the elasticity, which is called the cross-price elasticity of demand (for X with respect to Y):

$$\epsilon_{XY} = \frac{\% \Delta X}{\% \Delta P_Y} = \frac{\frac{\Delta X}{X}}{\frac{\Delta P_Y}{P_Y}} = \frac{\frac{dX}{X}}{\frac{dP_Y}{P_Y}} = \frac{dX}{dP_Y} \frac{P_Y}{X}$$

This leads us to one more set of classification of goods by their cross-price elasticity

$\epsilon_{XY} > 0$ means X is a substitute for Y

$\epsilon_{XY} = 0$ means X and Y are unrelated goods

$\epsilon_{XY} < 0$ means X is a complement to Y

e.g.: http://en.wikipedia.org/wiki/Cross_elasticity_of_demand

Good	Good with Price Change	XED
Butter	margarine	+0.81
Beef	Pork	+0.28
Entertainment	Food	-0.72

note also that in all these cases the sign of the elasticity is determined by the sign of the slope of the underlying function (because quantities and prices have to be positive)

So we could write for each person a demand function:

$$Q_X = f(P_X, P_Y, I)$$

I generally distinguish a demand function from a demand curve, which just shows Q and (its own) P

The demand curve essentially holds everything else constant and varies only price changes in income or in prices of other goods therefore cause shifts of the demand curve rather than movements along it

we'll confine our manipulations next lecture to this function

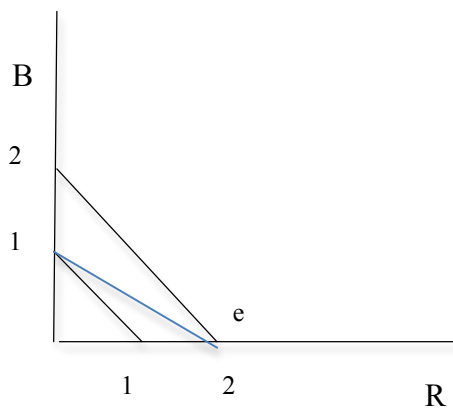
Answers to Practice Problems from 2/2/11

I. 1) $MRS = 2\sqrt{Y}$; it depends only on Y and not on X

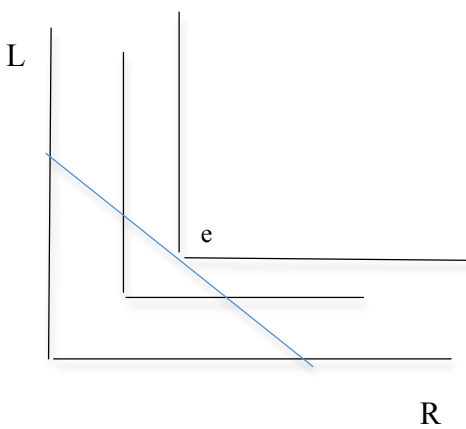
2) $2X + Y = 41$; can rearrange to $Y = 41 - 2X$; slope = -2

3) $2\sqrt{Y} = 2$, so $Y = 1$ and thus $X = (41-1)/2 = 20$, so (20,1)

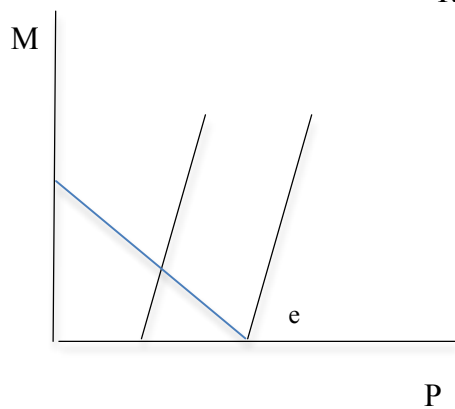
II. 1)



2)



3)



Practice Problems 2/4/11

I. Consider the following demand function:

$$X = \frac{I}{2P_X P_Y}$$

Assume that $I = 20$, $P_X = 2$, and $P_Y = 1$

- 1) What is the formula for the income elasticity of demand?
[note. To answer these problems, substitute in the two numbers that you don't care about before differentiating; for instance in this case substitute in for P_X and P_Y , leaving I as a variable]
- 2) What is the value for the income elasticity of demand? What type of good is X ?
- 3) What is the formula for the own price elasticity of demand? What is the value for the own price elasticity of demand? What type of good is X ?
- 4) What is the formula for the cross price elasticity of demand? What is the value for the cross price elasticity of demand? What type of good is X ?

II. Consider the following situations:

- 1) If the income elasticity of demand for restaurant meals is 1.40 and income declines by 10%, what happens to the quantity demanded of restaurant meals? What type of good are restaurant meals?
- 2) If the own price elasticity of demand in China is -0.8 and the price of rice rises by 20%, what happens to the quantity demanded of rice? What type of good is rice?
- 3) If the cross price elasticity of demand for beef with respect to pork is 0.28 and the price of pork declines by 50%, what happens to the quantity demanded of beef? What is the relationship between beef and pork?