

8th Class

2/7/11

go over problem answers from last time; no new problems today given you have your problem set to work on; we'll do some problems for these concepts later after we introduce parallel ones for the supply/producer side

finish our discussion of consumer theory

see handout on the full structure of the consumer demand function system

revisit the idea of experimenting with animals to see if they follow consumer theory as well; generate downward-sloping demand curves for instance?

[show classic papers on this topic]

animal paper 1 on the original rat experiment (show pp. 1, 3, 8)

<http://onlinelibrary.wiley.com/doi/10.1111/j.1465-7295.1975.tb01101.x/abstract>

note that prices in this case (root beer vs. collins mix) are number of lever presses required to get a drink

animal paper 2 summarizes a broader range of experiments on animals (show pp. 1, 14) that shows substitutability holds across a range of situations

<http://www.jstor.org/stable/2936137>

note that in the handout I didn't specify the exact functional form of the utility function; we've seen several cases. Three of the most used forms are:

quasilinear (which we have used): $U = u(X) + Y$; $u(X)$ is a subfunction; e.g. $U = \sqrt{X} + Y$

Cobb-Douglas (named after the two economists who first used it):

$$U = AX^aY^b ; A, a, \text{ and } b \text{ are constants (numbers); e.g. } U = 5X^2Y^3$$

note that is actually a special case of a more general function, Constant Elasticity of Substitution (CES):

$$U = A[aX^{\frac{s-1}{s}} + (1-a)Y^{\frac{s-1}{s}}]^{\frac{s}{s-1}}; A, a, \text{ and } s \text{ are constants}$$

In each of these cases we can analytically derive demand functions from the utility function once we also specify the budget constraint. For instance in the Cobb-Douglas case the demand functions take the form:

$$X = \frac{\frac{a}{a+b}I}{P_X} \quad \text{and} \quad Y = \frac{\frac{b}{a+b}I}{P_Y} \quad (\text{note the cross-price and } A \text{ don't appear})$$

individual and market demand curves (slope and elasticity)

recall movement along rather than shifts of the demand curve

show how to cumulate individual demand into market demand

e.g. two individuals have the following demand curves:

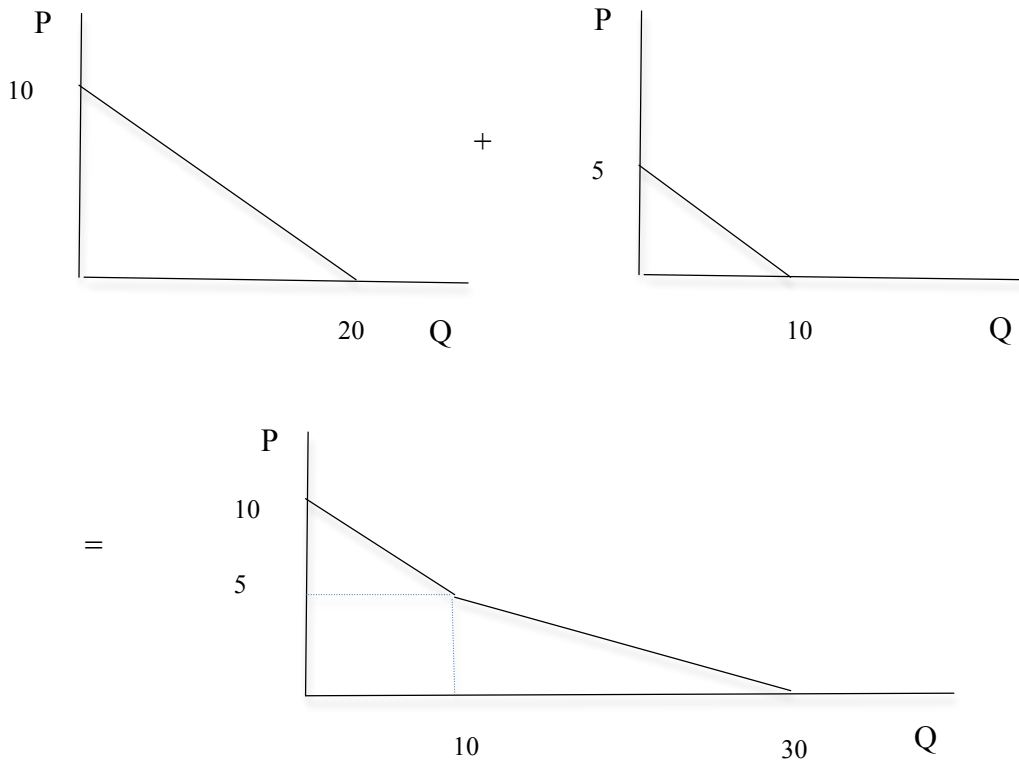
$$Q = 10 - 2P \quad (\text{which can be rewritten as } P = 5 - .5Q)$$

$$Q = 20 - 2P \quad (\text{or } P = 10 - .5Q)$$

so market demand is:

0	if $P < 0$ (by definition can't be negative)
$Q = 30 - 4P$	if $0 < P < 5$ (add the two curves in this section)
$Q = 20 - 2P$	if $5 \leq P \leq 10$ (only the person with greater demand)
0	if $P > 10$

graph them; note the kink point at (5,10) and then the slope gets flatter after that point (so slope on the top segment is $-.5$ and on the bottom segment is $-.25$)



note I was confounding the two last lecture when I showed you calculated elasticities; they were either averages across a set of individuals, or market rather than individual demand elasticities

think of examples of when individual demand (and thus market demand) would be either perfectly inelastic or perfectly elastic
 show how to cumulate individual into market demand function for those special cases (in these cases the slope of the market demand curve is the same as the slope of the individual demand curves)

discuss slope and elasticity at both levels, motivate why market is more elastic than individual

show slope and elasticity of a straight-line demand curve

$$\text{e.g. } Q = 100 - 10P, \text{ then } \frac{dQ}{dP} = -10, \epsilon = -\frac{10P}{Q} = -\frac{10P}{100 - 10P} = \frac{P}{P - 10}$$

then could evaluate at a particular (P,Q) point, e.g. if P = 5 then $\epsilon = -1$, but if P = 6 then $\epsilon = -1.5$

discuss constant elasticity curves, remind that elasticity changes if slope is constant so a constant elasticity demand curve must be curved

consumer surplus:

new way of thinking about the demand curve: indicates both total and marginal willingness to pay; area under the demand curve up to the level of quantity currently consumed is total willingness to pay, slope at the current chosen point is marginal willingness to pay

as you have more of the good you're willing to pay less for each marginal unit (remember there is the opportunity cost because you have to get less of something else if you're going to buy more of this good, given a certain level of income); i.e., there's diminishing marginal utility. Note this is sort of letting us translate utils into dollars.

consider a very simple problem where you can consider the utility separately from each good, and then look at a single good and the utility, or value, that it generates, $U(Q)$

a consumer facing price P and choosing to buy Q units of the good gets a net value, or consumer surplus, of $CS = U(Q) - PQ$

note that to maximize CS, set $\frac{dCS}{dQ} = \frac{dU}{dQ} - P = 0$, or $MU = P$

in the simple linear demand curve case, if you are consuming at P^*, Q^* , then

$$CS = \frac{1}{2}(P^{\max} - P^*)Q^*$$

using the formula for area of a triangle

more generally (which allows for curved demand curves, like constant elasticity ones):

$$CS = \int_0^{Q^*} (MU(x) - P)dx = \int_{P^*}^{\infty} Q(y)dy$$

(we won't be using the integral version, but I wanted you to see it)

Answers to Practice Problems from 2/4/11

I. 1) $\epsilon_I = \frac{I}{4X}$

2) $\epsilon_I = 1$; a normal good (with unit elasticity)

3) $\epsilon_{XX} = -\frac{10}{P_X X}$; $\epsilon_{XX} = -1$; an ordinary good (with unit elasticity)

4) $\epsilon_{XY} = -\frac{5}{P_Y X}$; $\epsilon_{XY} = -1$; a complement

II. 1) it declines by 14%; a luxury good

2) It falls by 16%; an ordinary good

3) It declines by 14%; beef is a substitute for pork

Handout on Demand Systems 2/7/11

Consider the utility function $U = U(X, Y)$

and the budget constraint $P_X X + P_Y Y = I$

Then there are 2 associated demand functions:

$$X = X(P_X, P_Y, I)$$

$$Y = Y(P_X, P_Y, I)$$

and 6 associated elasticities:

2 income elasticities:

$$\epsilon_{XI} = \frac{\% \Delta X}{\% \Delta I} = \frac{dX}{dI} \frac{I}{X}$$

$$\epsilon_{YI} = \frac{\% \Delta Y}{\% \Delta I} = \frac{dY}{dI} \frac{I}{Y}$$

2 own-price elasticities:

$$\epsilon_{XX} = \frac{\% \Delta X}{\% \Delta P_X} = \frac{dX}{dP_X} \frac{P_X}{X}$$

$$\epsilon_{YY} = \frac{\% \Delta Y}{\% \Delta P_Y} = \frac{dY}{dP_Y} \frac{P_Y}{Y}$$

2 cross-price elasticities:

$$\epsilon_{XY} = \frac{\% \Delta X}{\% \Delta P_Y} = \frac{dX}{dP_Y} \frac{P_Y}{X}$$

$$\epsilon_{YX} = \frac{\% \Delta Y}{\% \Delta P_X} = \frac{dY}{dP_X} \frac{P_X}{Y}$$

This framework can be generated to any number of goods (represented by 1, 2, ..., n):

Consider the utility function $U = U(X_1, X_2, \dots, X_n)$

and the budget constraint $P_1X_1 + P_2X_2 + \dots + P_nX_n = I$

Then there are n associated demand functions:

$$\begin{aligned} X_1 &= X_1(P_1, P_2, \dots, P_n, I) \\ X_2 &= X_2(P_1, P_2, \dots, P_n, I) \\ &\dots \\ X_n &= X_n(P_1, P_2, \dots, P_n, I) \end{aligned}$$

and $n*(n+1)$ associated elasticities:

n income elasticities:

$$\begin{aligned} \epsilon_{X_1I} &= \frac{\% \Delta X_1}{\% \Delta I} = \frac{dX_1}{dI} \frac{I}{X_1} \\ \epsilon_{X_2I} &= \frac{\% \Delta X_2}{\% \Delta I} = \frac{dX_2}{dI} \frac{I}{X_2} \\ &\dots \\ \epsilon_{X_nI} &= \frac{\% \Delta X_n}{\% \Delta I} = \frac{dX_n}{dI} \frac{I}{X_n} \end{aligned}$$

n own-price elasticities:

$$\begin{aligned} \epsilon_{X_1X_1} &= \frac{\% \Delta X_1}{\% \Delta P_1} = \frac{dX_1}{dP_1} \frac{P_1}{X_1} \\ \epsilon_{X_2X_2} &= \frac{\% \Delta X_2}{\% \Delta P_2} = \frac{dX_2}{dP_2} \frac{P_2}{X_2} \\ &\dots \\ \epsilon_{X_nX_n} &= \frac{\% \Delta X_n}{\% \Delta P_n} = \frac{dX_n}{dP_n} \frac{P_n}{X_n} \end{aligned}$$

$n*(n-1)$ cross-price elasticities:

$$\begin{aligned} \epsilon_{X_1X_2} &= \frac{\% \Delta X_1}{\% \Delta P_2} = \frac{dX_1}{dP_2} \frac{P_2}{X_1}; \epsilon_{X_1X_3} = \frac{\% \Delta X_1}{\% \Delta P_3} = \frac{dX_1}{dP_3} \frac{P_3}{X_1}; \dots; \epsilon_{X_1X_n} = \frac{\% \Delta X_1}{\% \Delta P_n} = \frac{dX_1}{dP_n} \frac{P_n}{X_1} \\ \epsilon_{X_2X_1} &= \frac{\% \Delta X_2}{\% \Delta P_1} = \frac{dX_2}{dP_1} \frac{P_1}{X_2}; \epsilon_{X_2X_3} = \frac{\% \Delta X_2}{\% \Delta P_3} = \frac{dX_2}{dP_3} \frac{P_3}{X_2}; \dots; \epsilon_{X_2X_n} = \frac{\% \Delta X_2}{\% \Delta P_n} = \frac{dX_2}{dP_n} \frac{P_n}{X_2} \\ &\dots \\ \epsilon_{X_nX_1} &= \frac{\% \Delta X_n}{\% \Delta P_1} = \frac{dX_n}{dP_1} \frac{P_1}{X_n}; \epsilon_{X_nX_2} = \frac{\% \Delta X_n}{\% \Delta P_2} = \frac{dX_n}{dP_2} \frac{P_2}{X_n}; \dots; \epsilon_{X_nX_{n-1}} = \frac{\% \Delta X_n}{\% \Delta P_{n-1}} = \frac{dX_n}{dP_{n-1}} \frac{P_{n-1}}{X_n} \end{aligned}$$