Problem Set #2

Due Wednesday 2/23/11 by 6 p.m. in the Econ 110-1 slot in the Economics Alcove

Use a stapler! Write legibly and use full and grammatically correct sentences in your answers. Draw graphs neatly and label axes and points clearly. Show your work on calculations. Each part below is weighted equally in grading, as are subparts within a part.

A. Manipulation of production functions and cost functions:

Consider the production function $Q = 3\sqrt{K} + 4\sqrt{L}$

where $K =$ units of capital and $L =$ units of labor

Assume that the firm takes the prices of capital ($r$) and labor ($w$) as given.

1) a. What is the marginal product of capital? What is the marginal product of labor?

   b. What is the MRTS?

2) a. Sketch two isocost curves for this firm. Draw an isoquant tangent to one of them.

   b. What is the slope of the isocost curves?

3) Assume that $w = 4$ and $r = 6$, and that the firm wants to produce 22 units of output.

   a. Solve for the optimal amounts of capital and labor that the firm should purchase.

   b. What is the firm’s cost of producing this level of output?

4) Assume that the wage rises to 8, while the rental rate of capital falls to 1. The firm still wants to produce 22 units of output.

   a. Solve for the optimal amounts of capital and labor that the firm should purchase now.

   b. Now what is the firm’s cost of producing this level of output?
B. Solving the firm’s problem:

Consider the cost function \( C = 20Q^2 - 40Q + 180 \)

where \( Q \) = the firm’s output

1) a. What is the firm’s VC? FC?
   b. What is the firm’s AVC, AFC, and AC?
   c. What is the firm’s MC?
   d. Sketch the firm’s AC, AVC, AFC, and MC curves all in the same graph.

2) Assume the firm takes the price of output (\( P \)) as given and equal to 120.
   a. Write down the firm’s profit function (as a function of \( Q \)).
   b. Solve for the firm’s optimal value of \( Q \) if they were to produce output.
   c. What is the firm’s TR at this point? What is the firm’s TC?
   d. What is the firm’s profit at this point? In the short run, should the firm operate, or shut down? In the long run, should the firm operate, or shut down?

3) Assume that the price of output falls to 40.
   a. Solve for the firm’s new optimal value of \( Q \). What is the firm’s profit at this point? In the short run, should the firm operate, or shut down? In the long run, should the firm operate, or shut down?
   b. Sketch the firm’s AC, AVC, MC curves and a line indicating price \( P \) all in the same graph. Indicate the firm’s optimal point of production in the graph and the associated amount of profit.

4) a. Solve for the firm’s long-run equilibrium price and quantity (i.e., the point at which its profit is equal to zero).
   b. Mark this point on your graph from 3)b.
C. Short-run and long-run equilibrium in a perfectly competitive market:

The market for wudgets is perfectly competitive.

There are 100 buyers, each with a demand curve of \( q = 4 - \frac{1}{2}P \).

There are 50 sellers, each with a cost function of \( c(q) = q^2 + 2q + 1 \).

1) a. What is the market demand curve? Graph it as well as writing down the appropriate algebraic statement.

b. What is the market supply curve? Graph it on the same graph as a. as well as writing down the appropriate algebraic statement.

2) a. What are the equilibrium price and quantity? Mark this point on your graph from (1).

b. How many units does each buyer purchase? How many units does each seller sell? Does economic theory tell us how to match buyers and sellers?

3) a. Calculate profits for a firm. Will there be any exit or entry occurring in this market?

b. If fixed costs rose to 4, what would happen in the market?

4) Suppose a tax of 3 is levied per wudget. Fixed costs are still 1.

a. Solve for the new equilibrium price(s) and quantity.

b. Calculate profits for a firm. Will there be any exit or entry occurring in this market?
D. Effects on economic efficiency of market interventions:

The weekly demand of an island community for gallons of heating oil is \( Q_D = 6000 - 300P \).

The price of oil is set at $5 per gallon by the island’s government.

The domestic supply of oil is fixed (perfectly inelastic) at 3000 gallons.

An additional quantity of oil is imported, which allows demand to be met at the set price.

The community has decided to discontinue importing oil. The town council is choosing between the options of rationing the domestic supply of oil at $5, or allowing the price to rise to the market-clearing level.

1) a. How much oil is demanded at the set price?

b. At what price would demand equal domestic supply?

c. What would be the weekly change in consumer surplus caused by raising the price to this level (compared to continuing to import)?

d. What would be the weekly change in consumer surplus from running a rationing program (compared to continuing to import)?

2) Draw a graph of the island market for heating oil.

a. Indicate on the graph the area of consumer surplus that would be lost by stopping imports and allowing price to rise freely.

b. Indicate on the same graph the area of consumer surplus that would be lost (the deadweight loss) by initiating a rationing program while keeping the price at $5.

c. Indicate on the same graph the area of consumer surplus that would be transferred to domestic oil suppliers under the market-clearing price option.

d. Indicate on the same graph the area of consumer surplus that is freed up for consumers to spend on other goods under either option.

3) Indicate which program you, as an outside consultant, would recommend that the community initiate, and explain your choice using economic reasoning.

4) Can you think of a better way to reduce the community’s dependence on imported oil? Discuss some possible alternatives.