

**Answers to Midterm**

- (1) a. Discuss whether results would be biased upwards or downwards based on your view of how memory reliability works. Not obvious that there would be bias overall in the sample.
- b. Discuss how these two factors bias the estimates: different for the two groups, or in the same direction for the two groups?

- (2) a.  $10\% + 20\% - 5\% = 25\%$
- b.  $10\% + 20\% + 20\% - (5\% + 5\% + 10\%) + 1\% = 31\%$
- c.  $(20\% - 5\% - 10\% + 1\%) / 31\% = 6\% / 31\% = 0.1935$ , or 19.35%

- (3) a. the probability of a true positive:

$$\Pr(\text{positive} \mid \text{positive}) = \frac{.8 * .01}{.8 * .01 + .1 * .99} = 0.0748, \text{ or } 7.48\%$$

- b.  $\Pr(\text{two "positives"} \mid \text{negative}) = (.1) * (.1) = .01$ , or 1%  
 $\Pr(\text{two "positives"} \mid \text{positive}) = (.8) * (.8) = .64$ , or 64%

- (4) a.  $E(\text{donut size}) = 0.5 * 4 + 0.5 * 8 = 6$  ounces  
 $\text{Var}(\text{donut size}) = 2 * 0.5 * (2 \text{ ounces})^2 = 4 \text{ ounces}^2$ , so Std. Dev. = 2 ounces

- b. Could show these in two sets (can be in two by two tables):

$$\begin{array}{ll} \Pr(16 \text{ ounce coffee} \mid 4 \text{ ounce donut}) = .40 & \Pr(24 \text{ ounce coffee} \mid 4 \text{ ounce donut}) = .60 \\ \Pr(16 \text{ ounce coffee} \mid 8 \text{ ounce donut}) = .80 & \Pr(24 \text{ ounce coffee} \mid 8 \text{ ounce donut}) = .20 \end{array}$$

$$\begin{array}{ll} \Pr(4 \text{ ounce donut} \mid 16 \text{ ounce coffee}) = .33 & \Pr(4 \text{ ounce donut} \mid 24 \text{ ounce coffee}) = .75 \\ \Pr(8 \text{ ounce donut} \mid 16 \text{ ounce coffee}) = .67 & \Pr(8 \text{ ounce donut} \mid 24 \text{ ounce coffee}) = .25 \end{array}$$

- c. There is more weight on the off-diagonal than on the main diagonal, so donut and coffee size are negatively correlated.

- (5) Grades depend on your examples being clear and being good illustrations of the phenomena

- (6) Good answers would have at least three categories of examples.

- (7) a. The null hypothesis is that there is no difference between the oxygen levels above and below the farm ( $H_0 : \mu_1 = \mu_2$  (or  $\mu_1 - \mu_2 = 0$ ); (can include that the below-level is greater than the above-level), while the (one-sided) alternative ( $H_a : \mu_1 > \mu_2$  (or  $\mu_1 - \mu_2 > 0$ ) is that the oxygen level below the farm is lower than above the farm. We get a difference of .18 (= 4.92 – 4.74) and a t-statistic of 1.96, with a p-value  $\Pr(T > t) = 0.0303$ ; i.e. there is only about a three percent probability that we would get this large of a difference if the null were true. So we fail to reject the null at the 1% of significance, but reject it at the 5% level.
- b. The null hypothesis is that the water quality level is greater than 4.75, and the alternative is that it is less than or equal to 4.75. Note you could have interpreted this question as asking about the water quality in general or just about the water quality below the farm. If in general, the mean is 4.83 and you cannot reject the null. If below the farm only, the mean is 4.74 and the t-stat is 0.12 and you cannot reject the null here either.
- (8) Answer depends on the specifics of your data.