(1) On Jan. 1, 2010, a liberal arts college installs 3000 new fire alarms. The time until a repair is first needed is specified by the installing company to be normally distributed with a mean of 365 days and a standard deviation of 100 days. The installing company guarantees that if the number of alarms needing replacement exceeds 150 within six months of the installation date (on or before July 1, 182 days later), the company will replace all the defective alarms free of charge.

a. What is the probability that any given alarm will have to be replaced on or before July 1?

b. What is the expected number that will have to be replaced on or before July 1?

c. What is the probability that the guarantee will be invoked?
(2) Suppose a magazine’s readership consists of two classes of people: 50% are poor, each earning $30,000 annually; 50% are rich, each earning $90,000 annually. A market research questionnaire insert asks, among other things, for the reader’s income. But each poor person is twice as likely to answer as each rich person, so the sample mean income is a biased estimator of the readership mean income. If a sample of 400 replies is obtained:

a. What is the bias of \( \bar{X} \) ?

b. What is the MSE of \( \bar{X} \) ?
A random check on election results in two towns yielded the following data (Note: these are true figures with some rounding to simplify the calculations):

East Haven: 54% out of the 9000 voters voted for the incumbent mayor
Middletown: 52% out of the 8500 voters voted for the incumbent mayor

Using appropriate p-values, evaluate the following claims made by a journalist reporting on these two elections:

a. “The election clearly shows that, in Middletown, a majority of the population favors keeping the incumbent mayor.”

b. “The elections clearly show that a larger proportion of the population favors keeping the incumbent mayor in East Haven than in Middletown.”
(4) It is routine to administer a tuberculosis test, consisting of a skin reaction test, to American children as part of their annual pediatric examination. The skin reaction test relates to the width of the bump that appears on the skin where the shot is administered. Assume the diameter of the bump for a healthy individual ranges from 0 to 10 mm, following a uniform distribution. The diameter of the bump for a tubercular individual ranges from 2 to 12 mm, following a uniform distribution.

a. For this test, what are the null and alternative hypotheses? What are the Type I and Type II errors?

b. If the pediatrician sets the cutoff point for the test at 8 mm, what is the probability of Type I error? What is the probability of Type II error?

c. Concerned about the error rates, a doctor suggests that children take two consecutive reaction tests. Assume that tests are independent. If a person is only diagnosed as tubercular if both tests come in over 8 mm, now what is the probability of Type I error? Of Type II error?
A marketing firm is making a presentation to you, the owner of a small business, to convince you to hire them to increase your sales. It shows you the following data from five of its current clients:

<table>
<thead>
<tr>
<th>Client</th>
<th>2008 Revenue (in Million$)</th>
<th>2007 Marketing budget (in Million$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>26</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>38</td>
<td>12</td>
</tr>
</tbody>
</table>

a. Calculate the appropriate regression line. What null hypothesis are you implicitly getting ready to test?

b. What about this makes you wonder whether the firm may simply be making these numbers up?

c. The firm suggests that you budget $18 million for marketing and says that will lead to expected revenues in the following year of $55 million. Putting aside your concern about this firm’s reliability as expressed in your answer to b, for what additional reason should you be suspicious about this statement?