

Answers to Test #1

(1) many possible answers involving nonrandom selection

(2) a. **60 oz.**

b. **69 oz.**

c. **66 oz.**

d. 21.21, or **21 oz.**

(3) $\Pr(\text{passing at least one}) = \Pr(\text{pass math}) + \Pr(\text{pass Italian}) - \Pr(\text{pass both})$

$$= \Pr(\text{pass math}) + \Pr(\text{pass Italian}) - \Pr(\text{pass math}) \cdot \Pr(\text{pass Italian}) \quad [\text{independence assumption}]$$

$$= .75 + .85 - .75 \cdot .85 = 1.60 - .6375 = .9625, \text{ or } \mathbf{96\%}$$

You can also calculate this as $\Pr(\text{passing at least one}) = 1 - \Pr(\text{passing neither})$

$$= 1 - \Pr(\text{fail math}) \cdot \Pr(\text{fail Italian}) = 1 - .25 \cdot .15 = 1 - .02 = .9625, \text{ or } \mathbf{96\%}$$

(4) a. $\mu_X = n\pi = 8(.2) = \mathbf{1.6}$; $\sigma_X = \sqrt{n\pi(1-\pi)} = \sqrt{8(.2)(.8)} = \mathbf{1.13}$

b. $\mu_Y = 10\mu_X + 20 = 10 \cdot 1.6 + 20 = \mathbf{36}$; $\sigma_Y = 10\sigma_X = 10(1.13) = \mathbf{11.3}$

c. since $Y = 10X + 20$, $X = (50 - 20)/10 = 3$; from Table IIIc, $\Pr(X \geq 3) = \mathbf{.203}$

(5) a. $(7/8)^{25} = .035498$, or **3.55%**

b. $(1/8)^2 = .015625$, or **1.56%**

c. $\Pr(\text{winning at least twice}) = 1 - \Pr(\text{not winning}) - \Pr(\text{winning once})$

$$= 1 - .035498 - 25 \cdot (1/8) \cdot (7/8)^{24} = 1 - .035498 - .1267778 = .8377, \text{ or } \mathbf{83.77\%}$$

(6) Consider the first four games from the position of team A. In order for this to happen, they either win all four, or team B wins all four, which is equivalent to team A losing all four.

There are 16 possible sequences of wins and losses for team A for a four-game series:

WWWW

WWWL

WWLW

.

.

.

LLLW

LLLL

If the teams are equally matched, all sequences are equally likely,

so $\Pr(\text{four-game sweep}) = 2/16 = 1/8 = .125$, or **12.5%**

(7) This is an application of Bayes' Theorem:

$$\begin{array}{r}
 \begin{array}{l}
 |----- .95 \text{ "honest"} -----> = .912 \\
 .96 \text{ honest ----} \\
 |----- .05 \text{ "liar"} -----> = .048
 \end{array} \\
 \\
 \begin{array}{l}
 |----- .97 \text{ "liar"} -----> = .0388 \\
 .04 \text{ liar ----} \\
 |----- .03 \text{ "honest"} -----> = \underline{.0012}
 \end{array}
 \end{array}$$

100% of possible states of the world

a. $\Pr(\text{"honest"}) = .912 + .0012 = 0.9132$, or **91.32%**

b. $\Pr(\text{liar} | \text{"liar"}) = \frac{.0388}{.0868} = 0.4470$, or **44.7%**

(8) These can all be solved using the binomial probability tables with $n = 12$ and $\pi = .60$

a. $s = 8$ using the individual table, so $\Pr(8 \text{ hits}) = 0.213$, or **21.3%**

b. $s = 0$ using the individual table, so $\Pr(0 \text{ hits}) = 0.00$, or **0%**

c. $s = 2$ using the cumulative table, so $\Pr(\text{at least two hits}) = 1.000$, or **100%**

(9) These are both solved using the standard normal probability table

a. $\Pr(X < 24) = \Pr(Z < \frac{24 - 30}{10}) = \Pr(Z < -.60) = \Pr(Z > .60) = .274$, or **27.4%**

b. from Table IV, $.03 = \Pr(Z > 1.88) = \Pr(X > 10*(1.88) + 30) = \Pr(X > 48.8)$

so 48.8 months (or **4 years** with downward rounding)

(10) a. $E(\text{salad size}) = 0.5*10 + 0.5*20 = \mathbf{15 \text{ ounces}}$

$\text{Var}(\text{salad size}) = 2*0.5*5^2 = 25 \text{ ounces}^2$, so Std. Dev. = **5 ounces**

b. You just have to show one case in which multiplying the two marginal probabilities together does not equal the joint probability, e.g.:

$\Pr(12 \text{ oz latte}) * \Pr(10 \text{ oz salad}) = .4 * .5 = .20 \neq \Pr(12 \text{ oz latte and } 10 \text{ oz salad}) = .25$

Alternatively you can show that the conditional probability does not equal the unconditional probability for some outcome, e.g. $\Pr(12 \text{ oz latte} | 10 \text{ oz salad}) \neq \Pr(12 \text{ oz latte})$

c. Given dependence as shown in (b) and the pattern that there is more weight on the main diagonal than the off-diagonal, you can see that latte size and salad size are positively correlated.

$E(\text{salad size} | 12 \text{ ounce latte}) = (5/8)*10 + (3/8)*20 = \mathbf{13.75 \text{ ounces}}$

$E(\text{salad size} | 16 \text{ ounce latte}) = (5/12)*10 + (7/12)*20 = \mathbf{15.83 \text{ ounces}}$