

Answers to Test #1

(1) many possible answers involving nonrandom selection

(2) a. $93 - 21 = 72$ oz.

b. **68 oz.**

c. **65.8 oz.**

d. 21.78, or **22 oz.**

(3) a. $206/717 = .2873$, or **28.73%**

b. $109/717 = .1520$, or **15.2%**

c. $109/368 = .2962$, or **29.62%**

(4) $(0.44)^2 + (0.42)^2 + (0.10)^2 + (0.04)^2 = 0.3816$, or **38.16%**

(5) a. This problem is a binomial formula problem. With $\pi = \frac{1}{6}$, $n = 3$, and $s = 0$:

$$p(0) = \left(\frac{5}{6}\right)^3 = 0.579, \text{ or } 57.9\%$$

b. By repeated application of the binomial formula, you can calculate the other probabilities:

$$p(1) = \binom{3}{1} * \frac{1}{6} * \left(\frac{5}{6}\right)^2 = 0.347$$

$$p(2) = \binom{3}{2} * \left(\frac{1}{6}\right)^2 * \frac{5}{6} = 0.069$$

$$p(3) = \left(\frac{1}{6}\right)^3 = 0.005$$

The expected value of betting \$1 is $0.579*0 + 0.347*\$1 + 0.069*\$2 + 0.005*\$10 = \0.535
So you would lose an average of 46.5 cents for every dollar bet! A real sucker game.

(6) You will want to choose HT. If you instead pick TT, the only way you can win is if the first two tosses are both tails, which occurs with probability 0.25. If this does not happen, the next TT must be preceded by an H, and so I would already have won. So by choosing HT you have a 0.75 probability of winning.

(7) This is an application of Bayes' Theorem:

	.90 not defective	= .765
.85 adjusted	.10 defective	= .085
	.60 not defective	= .090
.15 not adjusted	.40 defective	<u>= .060</u>
		100% of possible states of the world

a. $\Pr(\text{defective}) = .85 \cdot .10 + .15 \cdot .40 = .085 + .060 = .145$

b. $\Pr(\text{adjusted} \mid \text{defective}) = \frac{.085}{.145} = 0.59$

(8) These are binomial problems and can be solved using Tables IIIb and IIIc or using the binomial formula with $n=9$, $s=0$ or 1 depending on the problem, and $\pi=.2$

a. $\Pr(\text{no pucker}) = .134$

b. $\Pr(\text{at least one pucker}) = 1 - .134 = .866$

c. $\Pr(\text{one pucker}) = .302$

(9) These are both solved using the standard normal probability table

a. $\Pr(X < 12) = \Pr\left(Z < \frac{12 - 20}{4}\right) = \Pr(Z < -2) = \Pr(Z > 2) = .023$, or **2.3%**

b. from Table IV, $.04 = \Pr(Z > 1.75) = \Pr(X > 4 \cdot (1.75) + 20) = \Pr(X > 27)$

so **27 months**

(10) a. $E(\text{milkshake size}) = 0.5 \cdot 16 + 0.5 \cdot 32 = \mathbf{24 \text{ ounces}}$

$\text{Var}(\text{milkshake size}) = 2 \cdot 0.5 \cdot 64 = 64 \text{ ounces}^2$, so Std. Dev. = **8 ounces**

b. $E(\text{milkshake size} \mid 8 \text{ ounce hamburger}) = \frac{1}{3} \cdot 16 + \frac{2}{3} \cdot 32 = \mathbf{26.67 \text{ ounces}}$

c. **No.** From observation, the rows and columns are not proportional to each other in the table. A quick calculation verifies dependence: $\Pr(4,16) = 0.30 \neq \Pr(4) \cdot \Pr(16) = .40 \cdot .50 = 0.20$. You also know because the conditional expected value of milkshake size was not equal to the unconditional expected value.