

Answers to Test #2

$$(1) \text{ a. } \Pr(\text{alarm life} \leq 182 \text{ days}) = \Pr\left(Z < \frac{182 - 365}{100}\right) = \Pr(Z < -1.83) = \Pr(Z > 1.83) = 0.034,$$

so there is a **3.4%** probability that any given alarm will fail.

b. The expected number of failures is given by $3000 \cdot 0.034 = \mathbf{102}$ alarms.

c. $\Pr(\# \text{ of failures by July 1} \geq 150) = \Pr(\text{the sample proportion failing} > 150/3000) = \Pr(P > 0.05)$

$$= \Pr\left(Z > \frac{.05 - .034}{\sqrt{\frac{(.034)(.966)}{3000}}}\right) = \Pr(Z > 4.7) = 0.0000013, \text{ or } \mathbf{0.00013\%}$$

$$(2) \text{ a. } E(\bar{X}) = \frac{2}{3} * \$30,000 + \frac{1}{3} * \$90,000 = \$50,000$$

$$\mu = .5 * \$30,000 + .5 * \$90,000 = \$60,000$$

$$\text{then bias} = E(\bar{X}) - \mu = \$50,000 - \$60,000 = \mathbf{-10,000}$$

$$\text{b. calculate } \sigma_X^2 = \frac{2}{3} * (30,000 - 50,000)^2 + \frac{1}{3} * (90,000 - 50,000)^2 = 800,000,000$$

$$\text{and } \text{Var}(\bar{X}) = \frac{\sigma_X^2}{n} = 800,000,000/400 = 2,000,000$$

$$\text{then MSE} = \text{Var}(\bar{X}) + \text{bias}^2 = 2,000,000 + 100,000,000 = \mathbf{102,000,000}$$

(3) a. Here, $H_0: \pi \leq .50$ and $H_A: \pi > .50$, where π = proportion voting for the incumbent

$$\text{SE}(P_M) = \sqrt{\frac{(.52)(.48)}{8500}} = .0054$$

$$t_M = \frac{.52 - .50}{.0054} = 3.7$$

$$\Pr(Z > 3.7) = \mathbf{.000108}$$

This one-sided p-value is essentially equal to 0; i.e., the value is so far out in the tail of the null hypothesis distribution that we can almost certainly reject the null hypothesis, in effect accepting the journalist's statement.

(3) b. $H_0 : \pi_E - \pi_M \leq 0$ and $H_A: \pi_E - \pi_M > 0$

$$SE(P_E - P_M) = \sqrt{\frac{(.54)(.46)}{9000} + \frac{(.52)(.48)}{8500}} = .0075$$

$$t = \frac{.54 - .52}{.0075} = 2.67$$

evaluate this using the standard normal table: $\Pr(Z > 2.67) = .004$

So the one-sided p-value is about .004, which passes at the 99.6% test level (i.e., reject the null).

(4) a. The null hypothesis is that the child does not have tuberculosis, the alternative hypothesis is that the child does have tuberculosis, a Type I error is a false positive, i.e. diagnosing tuberculosis when the child is healthy; and a Type II error is a false negative, i.e., failing to diagnose tuberculosis in an affected child.

b. $\alpha = \Pr(\text{Type I error}) = \mathbf{0.20}$

$\beta = \Pr(\text{Type II error}) = \mathbf{0.60}$

c. $\alpha = 0.20 * 0.20 = \mathbf{0.04}$

$\beta = 1 - (0.40 * 0.40) = 1 - 0.16 = \mathbf{0.84}$

(5) a. worksheet:

Y	X	$(X - \bar{X})$	$(Y - \bar{Y})$	xy	x^2
14	4	-3	-9	27	9
17	5	-2	-6	12	4
20	6	-1	-3	3	1
26	8	1	3	3	1
38	12	5	15	75	25
$\bar{Y} = 23$	$\bar{X} = 7$			$\sum xy = 120$	$\sum x^2 = 40$

$$b = \frac{\sum xy}{\sum x^2} = \frac{120}{40} = 3; a = \bar{Y} - b \bar{X} = 23 - (3)(7) = 2$$

so $Y = 2 + 3X$

I might be setting up to test either the null hypothesis that $\beta = 0$ (no effect of marketing on revenue); or $\beta \leq 0$ (a nonpositive effect of marketing on revenue)

b. All the data points lie exactly on the regression line, with no error! That is pretty suspicious.

c. This is an example of extrapolation, as the budget is well outside of the range of the x-values used in estimating the regression line, and thus is problematic.