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Economics 300
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Answers to Test #2

(1) a. Pr(average weight > 200) = $\Pr\left(Z > \dfrac{200-180}{\frac{30}{\sqrt{60}}}\right) = \Pr(Z > 5.16) \cong 0.0000001$,

so there is essentially **zero** probability of overloading.

b. Given the above answer, the airline should book **100%** —all 60—of its seats.

(2) a. bias(L) = E(L) - 15 = (.20*10 + .50*15 + .30*20) - 15 = 15.5 - 15 = 0.5, so **L is biased**

bias(W) = E(W) - 5 = (.20*(4 + .6) + .60*5) - 5 = 5 - 5 = 0, so **W is unbiased**

b. MSE(L) = $\text{Bias}^2 + \text{Var}(L) = (0.5)^2 + .20*(10 - 15.5)^2 + .50*(15 - 15.5)^2 + .30*(20 - 15.5)^2$

= 0.25 + 12.25 = **12.5**

c. For centered distributions, particularly ones with discrete values like this one, the mode does indeed provide a good estimate of the true value. For noncentered (non-bell-shaped) distributions, this would not be the case.

(3) a. set up the test statistic: $\dfrac{.52 - .60}{\sqrt{\dfrac{(.52)(.48)}{200}}}$ = 2.26, which rejects the null of 60% at the 5 percent level,

but not at the 1 percent level on two-tailed hypotheses tests

b. Using the standard normal table, Pr(2.26) = .012; doubled gives a p-value = .024

(4) a. the die may be fair with respect to threes but unfair with respect to at least two other sides of the die, e.g., it could be balanced to deliver a one 30% of the time and a two 10% of the time, but the other sides could still occur at the usual frequency of 20% each. Or the die could be balanced, but four sides could be labeled “1” and one side could be labeled “3”!

b. Assuming the die are imbalanced, but we do not know how, checking the relative frequency in the sample of one value relative to the fair die value of 20% appears a reasonable strategy. However, as the inspector does, we would want to allow for the sample proportion of threes to deviate somewhat from 20% without rejecting the die as it would naturally do this by chance even if it were a fair die.

c. While this is a binomial distribution problem, we can use the standard normal table to approximate the probabilities that we might get less than 3 or more than 7: the sum of these will be α. Note that I use the continuity correction to the binomial, so for “less than 3” I use 2.5 and for “more than 7” I use 7.5.

Note that $H_0 : \pi = .20$ and $H_A : \pi \neq .20$

under the null hypothesis, $SE(\pi) = \sqrt{\frac{(.2)(.8)}{25}} = .08$

$$\alpha = Pr(P < \frac{2.5}{25}) + Pr(P > \frac{7.5}{25}) = Pr(Z < \frac{.1 - .2}{.08}) + Pr(Z > \frac{.3 - .2}{.08}) = 2*Pr(Z > 1.25) = \mathbf{0.212}$$

(compare this estimate to the true α, calculated using the binomial formula for $n = 25$, $\pi = .2$, and s ranging from 3 to 7:

$$\alpha = 1 - \sum_{s=3}^{7} \frac{25!}{s!(25-s)!} (.2)^s (.8)^{25-s} = \mathbf{0.207}$$)

d. under the alternative hypothesis $H_A : \pi = .30$, $SE(\pi) = \sqrt{\frac{(.3)(.7)}{25}} = .092$

$$\beta = Pr(\frac{2.5}{25} < P < \frac{7.5}{25}) = 1 - Pr(Z < \frac{.1 - .3}{.092}) - Pr(Z > \frac{.3 - .3}{.092})$$

$$= 1 - Pr(Z < -2.17) - Pr(Z > 0) = 1 - Pr(Z > 2.17) - .5 = .5 - .015 = \mathbf{0.485}$$

note that the lower limit must be maintained, because if the inspector got a number less than 3, he would reject the die! (He has no idea whether the true probability is higher or lower than .2)

(compare this estimate to the true β, calculated using the binomial formula for $n = 25$, $\pi = .3$, and s ranging from 3 to 7:

$$\beta = \sum_{s=3}^{7} \frac{25!}{s!(25-s)!} (.3)^s (.7)^{25-s} = \mathbf{0.502}$$)

(5) worksheet:

Y	X	$(Y - \bar{Y})$	$(X - \bar{X})$	xy	x2
40	7	-12.4	-6.6	81.8	43.56
50	8	-2.4	-5.6	13.4	31.36
12	22	-40.4	8.4	-339.4	70.56
10	25	-42.4	11.4	-483.4	129.96
150	6	97.6	-7.6	-741.8	57.76

$\bar{Y} = 52.4$ $\bar{X} = 13.6$ $\sum xy = -1469.4$ $\sum x^2 = 333.2$

$$b = \frac{\sum xy}{\sum x^2} = \frac{-1469.4}{333.2} = -4.41; \ a = \bar{Y} - b\bar{X} = 52.4 - (-4.41)(13.6) = 112.376$$

so **Y = 112 - 4.4X**

You are likely setting up to test the null hypothesis that $\beta = 0$
(no effect of GDP per capita on infant mortality)

b. Predicted infant mortality rate = 112 – 4.4*10 = 112 - 44 = 68

You would want to test to see if the null holds, in which case one might not think there is a relationship at all, or at least not a very clearly-specified one. The line is being influenced very strongly by the Eretrea case (the only one above the mean).

As it happens, the t-statistic on b is –1.71, for a two-tailed p-value of 0.186. So the null hypothesis is not rejected.

Here is the plot of the data and the regression line: