

**Suggested Answers for Midterm**

1. the parameter; the estimator; the regression variance; the estimated regression variance; the variance of the estimator; the estimated variance of the estimator
2. a) The requirement of linearity means that intercept cannot be set independently of slope; one can think of the linearity as a constraint that uses up one degree of freedom. Alternatively, it is set as the value of Y when all the Xs are set to zero.  
b) This would be necessary to avoid bias; for instance if all values of X were the mean value then one would also always want to predict the mean value of Y as the outcome.
3. This can cause bias on the estimates of the parameters unless the omitted variable is uncorrelated with the included ones, which *a priori* is unlikely; the confidence intervals are also not the correct ones because the estimated variance of the parameters is affected; adding an irrelevant variable causes inefficiency but no bias
4. a)  $8 - 3 = 5$   
b) mean =  $17/3 = 5.67$ ; variance = 6.33 (s.d. = 2.52) [MSD = 4.22; sqrt = 2.05]  
c) Anything that wasn't flat out incorrect was considered sufficiently compelling by your annoying friend.  
d) As the sample size increases, estimates of all parameters of a function, including its overall shape, become more precise.
5. a)  $\beta = (400 - 100 \cdot 2) / (500 - 100 \cdot 4) = 2$ ,  $\alpha = 1 - 2 \cdot 2 = -3$ ,  $E(\text{dice}) = 10 \cdot E(\text{one throw}) = 10 \cdot 3.5 = 35$   
so guess is  $2 \cdot (-3) + 9 \cdot (2) + 35 = -6 + 18 + 35 = 47$   
b) Expected payoff is  $60 - V(W) = 60 - V(2\alpha_{OLS} + 9\beta_{OLS}) - V(\text{dice})$   
 $= 60 - 4V(\alpha_{OLS}) - 2 \cdot 2 \cdot 9 \cdot \text{Cov}(\alpha_{OLS}, \beta_{OLS}) - 81V(\beta_{OLS}) - 10 \cdot V(\text{one throw})$   
 $V(\alpha_{OLS}) = \sigma^2 \left( \frac{1}{100} + \frac{4}{100} \right) = \frac{5\sigma^2}{100}$ ,  $V(\beta_{OLS}) = \frac{\sigma^2}{100}$ ,  $\text{Cov}(\alpha_{OLS}, \beta_{OLS}) = \frac{-2\sigma^2}{100}$ ,  $V(\text{one throw}) = 2.92$   
so expected payoff =  $30.8 - 0.29\sigma^2$
6. They are both still unbiased estimates but now your slope estimate is an unbiased estimate of  $\beta/3$ .

$$7. \text{ a) } \hat{\delta} = \frac{\sum sy}{\sum y^2} = \frac{\sum (y-c)y}{\sum y^2} = 1 - \frac{\sum cy}{\sum y^2} = 1 - \hat{\beta}$$

b) True. The squared residuals are the same for both relationships.

c) False. The adjusted R-squared aspect is irrelevant since the two regressions have the same  $n$  and  $k$ . In comparing R-squareds, while SSR is the same, the variation in  $S$  is less than the variation in  $C$  (assuming the marginal propensity to consume is greater than .5), so SST is different.

8. False. Imposing any constraint inhibits the minimization of SSR and thus lowers  $R^2$ . In the theoretical best case, if the constraint were nonbinding the  $R^2$  would be unchanged.

9. For more background on the least absolute deviations estimator (LAD) and comparison of its pros and cons to those of OLS, I recommend the wikipedia article on this subject. The LAD estimator is more robust with respect to outliers, but it is possible that it may lead to multiple solutions which one may find unsettling. It is the MLE if errors are distributed by the Laplace distribution (which is kind of what you would get if you push in on a normal distribution from both sides). The relevant articles are linked to the course website.

10.  $SSR_P = 16(1.9)^2 = 57.76$  and  $SSR_1 = 15(1.7)^2 = 43.35$ ;

$F = [(57.76 - 43.35)/1]/(43.35/15) = 4.99$  with d.f. 1,15, which is significant at the 5% level; so yes