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Economics 385
Spring Semester 2014-15
March 4, 2015

Midterm

Each problem is weighted equally. In order to get full credit, you must show any calculations used to arrive at your answers and answer the questions completely.

1. Let $\hat{u}_i$ be the residuals from the OLS regression of y_i on x_i ($i = 1, 2, \ldots, n$). Derive the following results:

   a) $\sum_{i=1}^{n} \hat{u}_i = 0$

   b) $\sum_{i=1}^{n} x_i \hat{u}_i = 0$

   c) State in words the significance/implication of the second result.

2. Suppose you are estimating a model that explains aggregate consumer debt as a function of the level of interest rates. Would you rather sample data during a period of steadily rising interest rates, a period of stable interest rates, or a period of steadily declining interest rates? Discuss.

3. Suppose there is a population of cats on an isolated island for which you know that age is uniformly distributed. You randomly sample two cats from the population and find that one is 2 years old and one is 4 years old.

   a) What is the maximum likelihood estimate of the range of the cat age distribution for this population?

   b) You randomly draw a third cat from the population. Without looking at the cat, what is your best guess (i.e., the maximum likelihood guess) for its age?

   c) If the cat population instead had normally distributed ages, what is your best guess as to the third cat's age?

   d) If the cat population instead had normally distributed ages, what is your best guess (i.e., the maximum likelihood guess) for the variance of cat ages?

4. Suppose you are estimating a money demand function for a sample of 20 observations. The equation passes the Wald test, but fails the LR and LM tests.

   a) What does it mean to say that the equation passes or fails these tests?

   b) How can it be that the tests give different results?

5. For the model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + u$, for the two restrictions $\beta_1 = \beta_2 + 2\beta_3$ and $\beta_4 = 0$, write down the restricted equation that you would estimate.

6. The model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$ was estimated using OLS with 24 observations. The results were:

$$y = 4 + 7x_1 - 2x_2 + 4x_3$$
$$\quad\quad (3.5)\ (2.2)\ (0.9)$$

t-ratios are in parentheses and $R^2 = .95$

The same model was estimated with the restriction $\beta_2 = \beta_3$. The results were:

$$y = 2 + 5x_1 - 1.5(x_2 + x_3) \qquad R^2 = .80$$
$$\quad\quad (2.6)\ \ (1.8)$$

a) Test the significance of the restriction $\beta_2 = \beta_3$.
[Assume that any test statistic that you calculate that is over 5 rejects the null]

b) State the assumptions under which the test is valid.

c) If x_2 is dropped from the original equation, would $\bar{R}^2$ rise, fall, or stay the same?

d) If x_3 is dropped from the original equation, would R^2 rise, fall, or stay the same?

7. Suppose you want to investigate the effect of attending a charter school (as opposed to a regular public school) on students' test scores.

a) Explain what kind of data you would want to collect in order to perform this investigation and write down the equation that you would want to estimate. Make sure your notation is understandable.

b) Mention an omitted variable that you wish you could collect for this investigation but that is not likely to be available in any usual data set. Given that you do not have it, is your estimate of the coefficient on the charter school dummy biased upward or downward due to its absence?

c) What is the fundamental problem with using readily available data to estimate the effect of charter school attendance on test scores?

d) What would a researcher need to be able to do in order to solve this fundamental problem?

8. For each of the following statements, state whether it is true, false, or uncertain, and explain your answer.

a) If all of the X-variables in a regression are divided by 2, then all the OLS coefficients β_0 through β_k on the transformed data are also half the size of the coefficients in the original regression.

b) R^2 can never be less than 0.

c) t-statistics can never be less than 0

d) F-statistics can never be less than 0.