Equivalent Width

Clearly it is of interest to define a “strength” for an absorption feature that is independent of its shape. The accepted way to do this in astronomy is via the “equivalent width” which is defined as the width of a rectangular spectral feature whose height is equal to the height of the continuum and whose area is equal to the integrated area of the spectral feature in a flux versus wavelength plot.

\[ W_\lambda \equiv \int_0^\infty \frac{F_c - F_l}{F_c} d\lambda \]
Measurements of line strengths (normally, equivalent width) in astronomical objects can be used to derive the abundance of that element in the gas making up the stellar atmosphere (or nebula). Obviously, one only gets the abundance of the particular ionization state and excitation level that produces the line. The Boltzmann and Saha equations need to be applied then and the pressure and temperature of the gas known to derive an abundance of the element (i.e. of all ionization states and excitation levels).

**Curve of Growth:**
How does equivalent width depend on abundance of atoms? There are 3 regimes, the “linear” portion, the “saturation” portion, and the “wings”. Observations of many lines that arise from same lower level but have different oscillator strengths can be used to define the shape of the curve of growth.

*Figure 9.22* A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)
Cosmic Abundances:

The reference point for abundance determinations is generally taken to be
the “solar nebula”, i.e. the composition of the gas cloud out of which the sun and
solar system formed. That is because we can get the most detailed and precise
information about this cloud from spectral analysis of the Sun and from chemical
analysis of the most primitive meteorites (Carbonaceous Chondrites). It is
assumed that the present-day solar atmosphere generally reflects the original
abundance of the Sun as a whole, which in turn reflects the composition of the
original cloud. One exception is Li which is depleted during PMS stage by nuclear
burning.

Notes:
1) Hydrogen is the base line element for analyses in the Sun. Its abundance
comes from the continuum opacity though the H-minus ion.
2) Helium is very hard to measure in the Sun (no absorption lines are present). It
can be inferred from solar wind particles and from measurements in solar
chromosphere and corona. It can also be measured in much hotter stars.
3) Lithium gets depleted easily during PMS burning. It is only detectable at all
because it has a strong resonance line at 6707 angstroms, which is the equivalent
of the Ly-alpha line in HI. Strong Li in stars is an indicator that they are still quite
young (PMS stars).
4) Note the odd-even dichotomy of the higher elements. Even numbered elements have about a factor of ten higher abundance than their odd numbered neighbors.

5) Also note the relative peak of the “iron peak” elements, Fe and Ni.

6) Note that Iron-56 represents the most stable atom in the sense of the greatest binding energy per nucleon, so it is the dividing point between where nuclear reactions yield energy by fusion and where they yield them by fission.

7) For most stars, the pattern of abundance is the same as the Sun. The only thing that changes is the relative abundance of all the metals compared to H and He. In some cases, we get anomalous abundances, though, such as “carbon stars”, or “helium white dwarfs”, etc.
cosmic abundances of the elements (Anders & Ebihara, 1982)

- ● even Z
- ○ odd Z
We will return to the abundances of the elements after discussing the interiors of stars which are the site of element production (except for H, He, and Li which were produced in the Big BANG)
Full atmospheric models:

In addition to calculating the opacities correctly, there is one other physical principle employed in full atmospheric models -- the concept of hydrostatic equilibrium. This is simply that the pressure gradient in the atmosphere has to balance the force of gravity, or

\[
\frac{dP}{ds} = -g \rho \quad \text{or} \quad \frac{dP}{d\tau} = -\frac{g}{\kappa}
\]

This has the effect of adding an additional free parameter into the model atmosphere solution, namely the surface gravity of the star, \( g \), which you will recall is related to the mass (\( M \)) and radius (\( R \)) of the star by:

\[
g = \frac{GM}{R^2}
\]

Therefore, full atmospheric models are two parameter sets for (effective temperature, surface gravity) and they can be used to match the MK spectral type, which is also a two-parameter system. Of course, other dimensions can be added, in particular by allowing the chemical composition to vary.

Where they fail? -- hot stars because the assumption of LTE breaks down as the opacity becomes small and the atmosphere thick, and cool stars where opacity calculations are murder due to molecular bands and convective energy transport becomes important.
Stellar Interiors Preview:

This concludes our discussion of stellar atmospheres and in the second part of the course we move on to stellar interiors and evolution. There are many differences between these subjects. First, some new physics becomes important, including:

1) Convective (and conductive, in white dwarfs) energy transport becomes important
2) Nuclear Energy generation becomes important
3) Spherical symmetry is now the common geometry instead of plane parallel
4) Tests of the theory are relations between bulk stellar properties such as mass, luminosity, radius, effective temperature and age, rather than spectral energy distributions.
5) Also, the theory seeks to account for the production of the chemical elements that we see in the Universe, its time dependence and for many of the features of galaxies that we observe. Understanding stellar evolution, the birth and death of stars and how they interact with their environments is central to understanding the evolution of galaxies.