1. *Fear.* This problem shows how in the neoclassical long-run macro model, widespread fear across an economy will not cause a recession! This is an important and comforting insight for the long run, but on the other hand, in the long run we are all dead...

Suppose the production function for the one representative firm in the economy is $Y = f(L) = 20L^{4/5}$. This firm also has investment demand $I(r) = 0.8/r$.

There are $L = 40$ workers who inelastically supply labor. These workers collectively supply savings of $S = 10$ units of output. (Again, that’s total savings of all 40 workers.)

(a) Show that the labor demand curve is $L(w) = (16/w)^5$, graph the labor market, and show the equilibrium real wage. Also graph the capital market and show the equilibrium real interest rate.

(b) Verify the national income accounts identity, i.e. that income from wages and dividends (which equals consumption plus savings) equals output (consumption plus investment).

(c) Now suppose that people in this country hear about the U.S. Financial Crisis and worry that some horrible thing could happen to them too. Everyone becomes very fearful of the future. The consumers all increase their savings to $S' = 20$ to prepare for bad times ahead. The firm decreases its investment demand to $I(r) = 0.4/r$ due to pessimism. The firm also shifts down its labor demand curve to $L(w) = (8/w)^5$ – even though this is not profit maximizing because the production function remains unchanged. What happens to the real wage, real interest rate, income, and output?
2. *Botswana.* Let’s consider the IMF’s policy toward Botswana (criticized by Joseph Stiglitz in his book *Globalization and its Discontents*, pg. 38). Normalize Botswana’s working population to $L = 100$. Let Botswana initially have a production function $y = f(L) = 380L^{1/2}$, and assume the firms represented by this function are owned by the workers. Set the price of $y$ equal to 1.

(a) Find the equilibrium real wage in Botswana’s labor market and graph the labor market. Also graph the production function.

(b) Private Botswanans were saving 700. The government was spending 25% of GDP and collecting taxes of 25% of GDP. Firms’ investment demand function was $I = \frac{7630}{r}$. We will ignore foreign capital flows (actually, they were considerable in real life). Graph the capital market, showing the private, government, and total savings curves and the investment curve. What was the real interest rate?

(c) Stiglitz claims that Botswana faced two negative shocks in 1981 due to drought and problems in the diamond industry. We’ll model this by saying that the production function changed for the worse to $y = f(L) = 350L^{1/2}$. What was the new real wage and the new output?

(d) The government of Botswana collected taxes of 25% of the new, lower real GDP, but it did not take the IMF’s advice and continued to spend the same real amount as before the shocks. Real private saving and investment demand remained unchanged. Graph what happened in the capital market and find the new real interest rate.

(e) The neoclassical model suggests that real wages in Botswana will grow more slowly as a result of the government’s decision in (c). Why? What is an argument against this view?

Review Problems only, not to turn in:
3. *OldGermans.* In Germany, the birth rate is low and the population is ageing. As a result, the working age population is falling at about 0.2% per year. It has been suggested that this population decline puts the German economy at risk. This question asks you to use our simple neoclassical model to evaluate that claim.

Let there be $L = 243$ German workers who inelastically supply labor and who spend all of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. (Aggregate meaning we treat all the firms as if there were just 1.) Let $p = 1$.

(a) Find the equilibrium real wage in the labor market and graph the labor market.

(b) Verify that there is also equilibrium in the beer market and graph the production function. What share of workers' income comes from wages and what share from dividends?

(c) Suppose that over 10 years, the German population falls and there are only $L' = 198$ workers. Find the new general equilibrium.

4. *OldGermansSave.* As in OldGermans, there are 243 German workers who inelastically supply labor, but now they save 100 beers (in total) for the future and spend the rest of their income on beer consumption. These workers own the German beer firms which have aggregate production function $f(L) = \frac{54}{4}L^{4/5}$. The German beer firms have aggregate investment demand of $I = 1200/r$, where $r$ is the real interest rate. Let $p = 1$.

(a) Find the equilibrium real wage in the labor market and graph the labor market. Verify that there is also equilibrium in the beer market and graph the production function.

(b) Graph the capital market. What is the equilibrium real interest rate?
(c) What happens if Germans become more pessimistic and start saving 110 beers?

d) Go back to just 100 beers saved. What happens if the German government levies taxes of 60 beers but German Chancellor Angela Merkel drinks 80 beers?

Answer to Review Problems:

3. OldGermans a.

(a) We can find labor demand using $pMP_L = w$, so,

$$
\frac{4}{5} \cdot \frac{54}{4} L^{-1/5} = w \Rightarrow L^d = \left(\frac{54 \cdot 4}{5w}\right)^5
$$

Setting $L^d = L^s = 243$ gives an equilibrium real wage of $w = 14.4$.

(b) The total costs of the firm are $wL = 14.4 \cdot 243 = 3499.2$. The total revenues are $py = 1 \cdot f(243) = 4374$. Thus the profits, paid as dividends, are 874.8. The firm’s output is 4374.

Workers earn total wages of $wL = 3499.2$ and total dividends of 874.8. Their total consumption of beer is thus 4374, so there is equilibrium. 80% of the workers’ income is from wages, and 20% from dividends.

(c) $L^d$ is the same as before, but now setting $L^d = L^s = 198$, gives an equilibrium real wage of $w = 15$. Firm output is $f(198) = 3713$, of which the total costs are $wL = 2970$ and the dividends are 743.
Workers’ wages plus dividends sum to 3713, all of which they consume, so there is equilibrium in the goods market.

4. OldGermansSave\_a.

(a) We can find labor demand using $pMP_L = w$, so,

$$I \cdot \frac{4 \cdot 54}{5 \cdot 4} L^{-1/5} = w \Rightarrow L^d = \left( \frac{54}{5w} \right)^5$$

Setting $L^d = L' = 243$ gives an equilibrium real wage of $w = 3.6$. The total costs of the firm are $wL = 3.6 \cdot 243 = 874.8$. The total revenues are $py = 1 \cdot f(243) = 1093.5$. Thus the profits, paid as dividends, are 218.7. The firm’s output is 1093.5.

Workers earn total wages of $wL = 874.8$ and total dividends of 218.7. Their total consumption of beer is thus $1093.5 - 100 = 993.5$, and the remaining 100 beers are saved, so there is equilibrium.

(b) The equilibrium real interest rate is found by setting

$$I = S \Rightarrow \frac{1200}{r} = 100 \Rightarrow r = 12\%$$
(c) The higher savings reduces the real interest rate:

\[ I = S \Rightarrow \frac{1200}{r} = 110 \Rightarrow r = 10.9\% \]

(d) Consumption of beers falls to \(1093.5 - 100 - 60 = 933.5\). Private savings stays the same at 100. Government spending is 80, so \(T - G = -20\), i.e. the government runs a deficit. National saving is then \(100 - 20 = 80\). The real interest rate rises to

\[ I = S \Rightarrow \frac{1200}{r} = 80 \Rightarrow r = 15\% \]

Note that the government deficit fully crowds out private investment, which falls from 100 to 80.