ECON 110, Prof. Hogendorn, Spring 2009

Second Midterm Exam Answers

1. USChinaWages_a.

(a) The derivative of the production function, i.e. the marginal product of labor, is \( f'(L) = -2(L - 10) \). As long as \( L < 10 \), this is a positive number, so the production function slopes up. The second derivative is \( f''(L) = -2 \), which is negative, indicating that adding more labor decreases the marginal product. Hence, this is the case of diminishing returns to labor.

(b) Since both mills are perfect competitors, they will both set price times marginal product of labor equal to the wage. Using the formula for marginal product of labor from part (a), this is

\[
-2(L - 10) = w \Rightarrow L(w) = \frac{20 - w}{2}
\]

In China, we have \( L(0.57) = 9.715 \), while in the US we have \( L(11) = 4.5 \).

(c) False. Both mills have exactly the same production function, so for any given number of workers, the total and marginal product
is the same at both mills. It is true that the marginal product of labor in the US is higher than in China, but this is because the wage is higher in the US, so profit maximization dictates that a mill there should hire fewer workers than in China. Since there are diminishing returns to labor, fewer workers means higher marginal product.

That said, it is true that in the real world, the production function is not the same in the US and China. US workers generally have more physical and human capital to work with, so in real life, US workers in most industries really are more productive than the same number of Chinese workers working in China. This is the main reason that wages are so much higher in the US.

(d) We already found the labor demand curve in part (b), it is 
\[ L(w) = \frac{20-w}{2} \]. Elasticity of labor demand with respect to the wage is defined as
\[ \varepsilon = \frac{\% \Delta L}{\% \Delta w} = \frac{dL(w)}{dw} \frac{w}{L} \]

The derivative is \( \frac{dL}{dw} = -\frac{1}{2} \). Thus, in the US the elasticity of labor demand to the wage is \( -\frac{1}{2} \frac{11}{7} = -1.2 \) and in China the elasticity is \( -\frac{1}{2} \frac{0.57}{9.715} = -0.03 \). It makes sense that US labor demand is so much more elastic because diminishing returns have not set in nearly as much, and thus marginal productivity is very sensitive to the number of workers hired.

2. Fear-goods_a.

(a) Firms maximize profits by setting the marginal product of labor equal to the wage:
\[ f(L) = w \Rightarrow \frac{4}{5} 20L^{-1/5} = w \Rightarrow L^{-1/5} = \frac{w}{16} \Rightarrow L(w) = \left( \frac{16}{w} \right)^5 \]
(b) Since the labor market clears, employment is 40, and output is 

\[ Y = f(40) = 382.5. \]

Income is equal to wages plus dividends. Wages are \( w^\ell = 7.65 \cdot 40 = 306. \)

To find dividends, we need to find the profits of the firm:

\[ \Pi = pq - wL = 1 \cdot 382.5 - 306 = 76.5 \]

So total income is \( 306 + 76.5 = 382.5 \) which does indeed equal output.

(c) The labor market now equilibrates off the new, irrational labor demand curve, so

\[ \left( \frac{8}{w} \right)^5 = 40 \Rightarrow w = 3.83 \]

Not surprising, workers’ wages fall because the firms’ collective fear has essentially the same effect as if they all colluded to reduce wages. But since this is a neoclassical model, the labor market does still clear, and all 40 workers are still employed. That means that output is still \( Y = f(40) = 382.5. \)

It remains to be seen if there is really income to pay for this output. Total wages are now only \( 3.83 \cdot 40 = 153.2. \) But firm profits now rise (due to the lower labor costs) to \( 1 \cdot 382.5 - 153.2 = 229.3. \)

Therefore, dividends go up a lot, and total income is still \( 153.2 + 229.3 = 382.5, \) exactly enough to equal output.