Second Midterm Exam

Each part of a question (a, b, c, etc.) is worth 5 points. Make sure to allot your time accordingly. Total of 30 points, −1 for messiness.

1. **Obesity.** Suppose that utility for an individual's food consumption \( f \) and the average American's food consumption \( F \) is given by

\[
U(f, x, F) = \left( \frac{f^{1/2} - \frac{1}{2} F^{1/2}}{\frac{1}{2}} \right)^{3/2} + x
\]

The individual has a budget constraint of \( x + p f = m \).

(a) What is the individual's utility maximization problem? Assume the individual can control \( f \) and \( x \) but has no control over \( F \). What solution functions would emerge from solving this problem, and what would they indicate (just explain, no need actually to find the functions).

(b) Well, suppose you do want to find the solution functions. To avoid tedious algebra, you use Mathematica:

\[
\begin{aligned}
\text{In}[78]:& \quad \mathcal{L} = \left( \frac{\frac{1}{2} f^{1/2} - \frac{1}{2} F^{1/2}}{\frac{1}{2}} \right)^{3/2} + x - \lambda \star (x + p \star f - m) \\
\text{Out}[78]:& \quad 2 \sqrt{2} \left( \sqrt{f} - \frac{\sqrt{F}}{2} \right)^{3/2} + x - (-m + f \star p + x) \lambda \\
\text{In}[79]:& \quad \text{FOCS} = \{ D[\mathcal{L}, x] = 0, \ D[\mathcal{L}, f] = 0, \ D[\mathcal{L}, \lambda] = 0 \} \\
\text{Out}[79]:& \quad \left\{ 1 - \lambda = 0, \ \frac{3}{\sqrt{2}} \sqrt{f - \frac{\sqrt{F}}{2}} - p \lambda = 0, \ m - f \star p - x = 0 \right\}
\end{aligned}
\]
What does the line beginning “FOCs” do? Why is the first equals sign just single (=), but the next three equals signs are double (==)?

(c) The fact that $F$ appears in an individual’s utility function suggests that there is some social aspect to food beyond just individual food consumption. Explain in words what is going on and what it means for the socially optimal level of $f$.

(d) If all consumers have this utility function, write down the problem of a social planner who maximizes the utility of a representative consumer. Explain what is different about this problem from part (a), and what will be the effect on the solution for $f$. (Again, don’t solve out, just explain.)

2. *OverNet.* OverNet is an Internet company that had a promising patented technology. It entered the market with a monopoly over its product due to the patent.

(a) Write out the short run profit maximization problem that OverNet solves. You can assume output is $y$, demand is some function $p(y)$, and OverNet has total cost function $TC(y)$. Explain the condition that characterizes the solution to this problem.

(b) In the long run, two things change. First, OverNet can now vary the amount of capital, which was not possible in the short-run problem. Second, the patent expires so other firms enter the market and OverNet becomes a perfect competitor. Relative to part (a), what changes about the profit maximization problem OverNet solves? Can we say for sure whether OverNet’s new optimum quantity is higher or lower than in part (a)?

When you are finished, please keep the exam sheet and hand in your blue book. Thanks.