1. 

(a) The budget line pivots, with both the digital intercept falling and the vinyl intercept increasing. Since we know that the new consumption point is weakly up (i.e., not changed) and to the right of the old point, the consumer moves to a higher indifference curve. This is consistent with the utility maximizing model. Note that the budget line must pivot above the (4.5,118) point, so it must be that the price of vinyl decreased by more than the increase in price of digital.

(b) This case is similar, but now the budget lines pivot around the digital intercept. This makes it more clear that buying power went up (although in fact that is also true in the previous case). It is a little surprising that the consumer does not translate some of this additional buying power into purchase of digital. It seems the Engel curve for digital is flat –
the increase in real income has no effect on quantity of digital. This is possible, though, so there is no violation of the utility-maximizing model.

2. *DanBrown_a.*

(a) The written budget constraint is

\[ 15k + 18h = 20 \]

(b) The total differential is

\[ 15dk + 18dh = 0 \]

Solving for \( dh/dk \) gives

\[ \frac{dh}{dk} = -\frac{15}{18} \]
(c) The Pixlee MRS is
\[
MRS = -\frac{\text{MU}_k}{\text{MU}_h} = -\frac{0.9k^{-0.1}h^{0.1}}{0.1k^{0.9}h^{-0.9}} = -\frac{9}{k}h
\]
To find the optimum, set equal to the slope of the budget constraint:
\[
-\frac{9}{k} = -\frac{15}{18} \Rightarrow h = 0.093k
\]
Then substitute into the budget constraint to find the optimum consumption:
\[
15k + 18(0.093k) = 20 \Rightarrow k^* = 1.2, \quad h^* = 0.11
\]

(d) The Inkie MRS is
\[
MRS = -\frac{\text{MU}_k}{\text{MU}_h} = -\frac{0.5(k+1)^{-0.5}}{1} = -\frac{1}{2\sqrt{k+1}}
\]
This is always negative, so the indifference curves are downward sloping, but when \(k = 0\), the slope is \(-\frac{1}{2}\). That means the indifference curves do not asymptote to the vertical axis, instead they intersect it at a slope of minus one-half:

(e) The slope of the budget line is
\[
-\frac{15}{18} < -\frac{1}{2}, \text{ or more intuitively, } \frac{15}{18} > \frac{1}{2}
\]
Since \(1/2\) is as steep a slope as these indifference curves ever have, there cannot be a tangency point within the graph, and Inkies will spend all of their money on \(h\). Thus,
\[
h^* = \frac{20}{18} = 1.11, \quad k^* = 0
\]

(a)

$$MRS = -\frac{\partial u}{\partial G} \cdot \frac{\partial u}{\partial V} = -\frac{.9G^{-1}V^{1}}{.1G^{9}V^{-9}} = -\frac{9V}{G}$$

Now set MRS equal to the slope of the budget line:

$$-\frac{9V}{G} = -\frac{1}{.4} = -\frac{10}{4} \Rightarrow G = \frac{36}{10}V \Rightarrow G = 3.6V$$

So use 3.6 parts gin to one part vermouth.

(b)

$$MRS = -\frac{\partial u}{\partial G} \cdot \frac{\partial u}{\partial V} = -\frac{0.5\left(\frac{G}{25} - 1\right)^{-0.5}}{\frac{1}{25}} = -\frac{1}{50\left(\frac{G}{25} - 1\right)^{0.5}}$$

Set the MRS equal to the slope of the budget line:

$$-\frac{1}{50\left(\frac{G}{25} + 1\right)^{0.5}} = -\frac{10}{4} \Rightarrow \left(\frac{G}{25} - 1\right)^{0.5} = \frac{1}{125}$$

$$\Rightarrow \frac{G}{25} - 1 = \frac{1}{15,625} \Rightarrow G \approx 25$$

Since the utility function is quasilinear, the marginal utility of $V$ is always 1. The marginal utility of $G$ diminishes and gets too small to outweigh $V$ once $G = 25$. Provided there is enough income $m$ available, it is optimal to consume $G = 25$ and spend all remaining income on $V$.

(c) At $G = 25$, the total cost of gin is $25. Everything left over is spent on vermouth, so $V = \frac{m-25}{.40}$. Then the ratio is

$$\frac{G}{V} = \frac{25}{\frac{m-25}{.40}} = \frac{10}{m-25}$$