1. HouseBuying
   
   (a) The Slutsky equation is
   \[ \frac{\partial h}{\partial p_h} = \frac{dh^s}{dp_h} + (\omega_h - h^*) \frac{\partial h}{\partial m} \]
   The term \( \frac{dh^s}{dp_h} \) must be negative because it is a pure substitution effect. The term \( \omega_h - h^* \) must be negative because this is a move-up household. The term \( \frac{\partial h}{\partial m} \) is the slope of the Engel curve for housing. It makes sense to think that housing is a normal good, so this term is probably positive. If this is correct, then both the substitution effect and the income effect are negative, and so the whole expression is definitely negative.

(b) 

(c) The downsizing household has the same Slutsky equation as in part (a), and all the signs of the derivatives should be the
same. But the term $\omega_h - h^*$ is now negative since this household is a net seller of housing. This makes the income effect positive. If it is large enough, an increase in the price of housing could actually make this couple spend more on their new house, buying a fancier (though still smaller) house than if the price of housing had not gone up.

(d) First, if the term $\omega_h - h^*$ is close to 0, then the income effect doesn't matter. This could happen if you found that when people buy new houses, they don't change the size or fanciness of their house very much. (It would also help if none of the people in your study owned multiple houses.) Second, if the term $\frac{\partial h}{\partial m}$ is small, then the income effect would be less important. This would be true if when people get more income (from any source, house selling or otherwise) they tend not to put that extra money into nicer houses. Unfortunately, this may not be very true. Finally, if the substitution effect $\frac{dh}{dp_h}$ is very large, it might make the income effect unimportant by comparison. This would occur if people have very elastic demand for housing. Unfortunately, this is again not too likely since housing is a necessity. Probably, then, this is not a situation where it's okay to ignore income effects.

2. SubsInc_a. For good measure, here's the Lagrangian to start:

$$\max_{x,y,\lambda} L = \sqrt{x} + \sqrt{y} - \lambda(p_x x + p_y y - m)$$

$$\frac{\partial L}{\partial x} = \frac{1}{2\sqrt{x}} - \lambda p_x = 0$$
$$\frac{\partial L}{\partial y} = \frac{1}{2\sqrt{y}} - \lambda p_y = 0$$
$$\frac{\partial L}{\partial \lambda} = p_x x + p_y y - m = 0$$
Solving simultaneously we get:

\[ \lambda = \frac{1}{p_x \sqrt{x}} \]

\[ \lambda = \frac{1}{p_y \sqrt{y}} \]

\[ \lambda = \lambda \Rightarrow \frac{1}{p_x \sqrt{x}} = \frac{1}{p_y \sqrt{y}} \]

\[ \Rightarrow x = \frac{p_y}{p_x} y \]

\[ \frac{p_y^2}{p_x^2} y + p_y y - m = 0 \Rightarrow y(p_x, p_y, m) = \frac{m}{\frac{p_y^2}{p_x} + p_y} \]

\[ x(p_x, p_y, m) = \frac{m}{\frac{p_x^2}{p_y} + p_x} \]

\[ \lambda = \frac{1}{2} \sqrt{\frac{p_y^2}{p_x} + p_x} \frac{1}{p_x} \]

Now, to find the substitution and income effect, we first need to know the starting point:

\[ y^* = y(1, 1, 100000) = \frac{100000}{\frac{1}{1} + 1} = 50000 \]

This means the Slutsky compensated demand is

\[ y^s(p_y) = y(p_x, p_y, p_x^* + p_y y^*) = \frac{p_x x^* + p_y y^*}{\frac{p_x^2}{p_y} + p_y} \]

The derivative of the Slutsky compensated demand function is:

\[ \frac{\partial y^s}{\partial p_y} = \frac{\partial y}{\partial p_y} + \frac{\partial y}{\partial m} y^* \]

Here that is

\[ \frac{\partial y^s}{\partial p_y} = -m \left( \frac{p_y^2}{p_x} + p_y \right)^{-2} \left( \frac{2p_y}{p_x} + 1 \right) + \frac{1}{\frac{p_y^2}{p_x} + p_y} 50000 \]
The Slutsky equation rearranges the partial derivative like this:

\[
\frac{\partial y^s}{\partial p_y} = \frac{\partial y}{\partial p_y} - \frac{\partial y}{\partial m} y^*
\]

So here that is

\[-75000 = -50000 - 25000\]

The substitution effect is \(-50000\) and the income effect is \(-25000\).

3. **Meat-packing.** Here is Mathematica code to answer part (c) of this problem:

```mathematica
L = y^{3/2}

TConEplant = 3(y/6)^{3/2}

TC = 6TConEplant

MC = D[TC, y]

Q = 24 - 4*p^(1/2)

inverseQ = p/.Solve[y == Q, p][[1]]

TR = inverseQ*y

MR = D[TR, y] // Simplify
```
4. *Unionize.* Widget producers have conditional factor demand for labor $L_D = y^2$. Labor supply is $L_S = w$ where $w$ is the wage (i.e. an upward-sloping 45-degree line). Inverse demand for widgets is given by $p = 5 - y$.

(a) Since labor is the only factor for this firm, $TC(y) = wL_D = wy^2$. Then $MC(y) = 2wy$. The firm will maximize profits when $MC(y) = p$. Treating our firm as the representative firm,

$$MC(y) = p$$
$$2wy = 5 - y$$
$$(2w + 1)y = 5$$
$$y^* = \frac{5}{2w + 1}$$

Then the unconditional labor demand is $L_D(y^*) = \frac{25}{(2w+1)^2}$. Labor market equilibrium is thus

$$L_D(y^*) = L_S$$
$$\frac{25}{(2w+1)^2} = w$$
$$25 = w(2w+1)^2$$
$$w^* = 1.5$$

(I used Wolphram Alpha for that last step.)

(b) Since demand is $p = 5 - y$, $TR(y) = p(y)y = (5 - y)y$ and
\( MR(y) = 5 - 2y \). The monopoly sets

\[
\begin{align*}
MR(y) &= MC(y) \\
5 - 2y &= 2wy \\
5 &= 2(1 + w)y \\
y_M &= \frac{5}{2(1 + w)}
\end{align*}
\]

Then the unconditional labor demand is \( L_D(y_M) = \frac{25}{4(1 + w)^2} \).

Labor market equilibrium is thus

\[
\begin{align*}
L_D(y^*) &= L_S \\
25 &= \frac{25}{4(1 + w)^2} \\
25 &= 4w(1 + w)^2 \\
w_M &= 1.24
\end{align*}
\]

(Again, I used Wolphram Alpha for that last step.)

(c) As far as goods market equilibrium and labor demand, we’re back to part (a), so \( y^* = \frac{5}{2w+1} \), and \( L_D(y^*) = \frac{25}{(2w+1)^2} \). We need to find the total wages as a function of the amount of labor supplied, so first we need to find inverse labor demand:

\[
\begin{align*}
L_D &= \frac{25}{(2w+1)^2} \\
(2w+1)^2 &= \frac{25}{L} \\
2w+1 &= \frac{5}{\sqrt{L}} \\
w_D(L) &= \frac{5}{2\sqrt{L}} = \frac{1}{2} \\
TR(L) &= w_D(L)L = \frac{5\sqrt{L}}{2} - \frac{L}{2} \\
MR(L) &= \frac{5}{4\sqrt{L}} - \frac{1}{2}
\end{align*}
\]
The union can take the labor supply function $L_s = w$ as the "marginal cost" $w_s(L) = L$ of its members working, so it maximizes its "profits" with a labor supply of

\[
\begin{align*}
MR(L) &= w_s(L) \\
\frac{5}{4\sqrt{L}} - \frac{1}{2} &= L \\
L_u &= 0.85
\end{align*}
\]

(Yet again, Wolphram Alpha.) The labor market will clear at

\[
\begin{align*}
L_d &= 0.85 \\
\frac{25}{(2w + 1)^2} &= 0.85 \\
w_u &= 2.21
\end{align*}
\]

This union wage is considerably higher than the perfectly competitive wage, but it comes at the expense of reducing output in the economy.