1. Read the introduction. We have already seen spillovers in a formal model a little bit, in Aghion et al. But there they were assumed in a very simple way in the background. Here, spillovers take on a key role, changing the equilibrium among oligopolistic firms in (I think) “neck-and-neck” competition.

2. Read Section I. There are three games here. All of them are two stage games where the firms first choose R&Ds $x$ then choose quantities $q$. The method for solving a game like this is subgame perfect Nash equilibrium. First, you solve the subgame of choosing $q$ assuming the $x$'s are given. Then you plug those solutions into the profit function and solve the first-stage game of choosing the $x$'s.

3. Let’s take a special case to make the algebra much easier. On page 1133, let $a=1$, $b=1.2$, $A=0.2$, and leave $\beta$ as a parameter since it measures spillovers which are the topic here. Then the demand and cost functions can be rewritten

$$p = 1 - 1.2(q_1 + q_2)$$
$$C_i = (0.2 - x_i - \beta x_j)q_i$$

At the bottom of the page it says the R&D costs are quadratic. That means they can be written

$$f(x_i) = \frac{x_i^2}{2}$$

Let’s use the simple case of $\gamma = 1$ so the cost of R&D $x_i$ is simply $x_i^2$.

4. Now come over to page 1134. Using all the simplifications we just made, rewrite the profit functions $\pi_i$ and $\pi_j$. Take the first order conditions (the partials of $\pi_i$ with respect to $q_i$ and of $\pi_j$ with respect to $q_j$ both set equal
to zero). Solve these two simultaneously and show that the sub game equilibrium quantities are $q_i$ as given in the paper.

5. Now substitute these answers into $\pi_1$ and $\pi_2$ to find $\pi^*_1$ and $\pi^*_2$. Again, take first order conditions, this time with respect to $x_1$ and $x_2$. Verify that the simultaneous solution is $x^*_i$ as given.

6. Finally, use graphing software to draw graphs of $x^*_1$ and $q^*_1$ as functions of $\beta$, letting $\beta$ vary from 0 to 1 (no spillover to full spillover).

7. Read Section II on welfare. A good source of potential value-added is to continue the example above into the rest of the paper.