Problem Set 3 Answers

1. *Shy6.8.3 a.* We have to work backwards, starting with stage 3. At that point, \( q_1 \) and \( q_2 \) are given, and firm 3 maximizes:

\[
\max_{q_3} \pi_3 = (120 - q_1 - q_2 - q_3)q_3
\]

This gives first order condition

\[
\frac{\partial \pi_3}{\partial q_3} = 120 - q_1 - q_2 - q_3 - q_3 = 0
\]

and reaction function

\[
q_3(q_1, q_2) = 60 - \frac{1}{2}q_1 - \frac{1}{2}q_2
\]

In Stage 2, firm 2 maximizes its profits, expecting firm 3 to behave as we just derived:

\[
\max_{q_2} \pi_2 = (120 - q_1 - q_2 - q_3(q_1, q_2))q_3 = (60 - \frac{1}{2}q_1 - \frac{1}{2}q_2)q_2
\]

This gives first order condition

\[
\frac{\partial \pi_2}{\partial q_2} = 60 - \frac{1}{2}q_1 - \frac{1}{2}q_2 - \frac{1}{2}q_2 = 0
\]

and reaction function

\[
q_2(q_1) = 60 - \frac{1}{2}q_1
\]

In Stage 1, firm 1 knows that firm 2 will play as above. It also knows that firm 3 will go on to react according to

\[
q_3(q_1, q_2(q_1)) = 60 - 0.5q_1 - 30 + 0.25q_1 = 30 - 0.25q_1
\]
Thus, firm 1 maximizes

\[ \max_{q_2} \pi_2 = (120 - q_1 - (60 - 0.5q_1) - (30 - 0.25q_1))q_1 = (30 - 0.25q_1)q_1 \]

This gives first order condition

\[ \frac{\partial \pi_1}{\partial q_1} = 30 - 0.25q_1 - 0.25q_1 = 0 \]

and optimal Stackelberg leader quantity

\[ q_1 = 60 \]

Plugging this leader quantity back into the other reaction functions, we see that the other two firms produce

\[ q_2 = 30 \quad q_3 = 15 \]

The total market quantity is thus 105, and the market price is

\[ p = 120 - 105 = 15. \]

2. Grandmammy_a.

(a) With a huge factory, firm 1 solves

\[ \max_{q_1} \pi_1 = (10 - q_1 - q_2)q_1 - 2k_1 - 1q_1 \]

This gives first order condition

\[ \frac{\partial \pi_1}{\partial q_1} = 10 - q_1 - q_2 - q_1 - 1 = 0 \]

and reaction function

\[ q_1(q_2) = \frac{10 - q_2 - 1}{2} = 4.5 - \frac{1}{2}q_2 \]
(b) With \( \bar{k}_f = 0 \), firm 1 solves

\[
max_{q_1} \pi_1 = (10 - q_1 - q_2)q_1 - 3q_1
\]

This gives first order condition

\[
\frac{\partial \pi_1}{\partial q_1} = 10 - q_1 - q_2 - q_1 - 3 = 0
\]

and reaction function

\[
q_1(q_2) = \frac{10 - q_2 - 3}{2} = 3.5 - \frac{1}{2}q_2
\]

(c) ABB always has MC=2. Since its problem is otherwise the same, we know that its reaction function will be

\[
q_2(q_1) = \frac{10 - q_1 - 2}{2} = 4 - \frac{1}{2}q_1
\]

(d) For the huge factory case, solving the reaction functions simultaneously gives

\[
q_1 = 4.5 - \frac{1}{2}(4 - \frac{1}{2}q_1) \Rightarrow \frac{3}{4}q_1 = 4.5 - 2 \Rightarrow q_1 = 3.33
\]

Then \( q_2 = 2.33, p = 10 - 3.33 - 2.33 = 4.33 \), and ABB’s profit is \( \pi_2 = (4.33 - 2)\times2.33 - 7 = -1.6 \).

For the small factory case, solving the reaction functions simultaneously gives

\[
q_1 = 3.5 - \frac{1}{2}(4 - \frac{1}{2}q_1) \Rightarrow \frac{3}{4}q_1 = 3.5 - 2 \Rightarrow q_1 = 2
\]

Then \( q_2 = 3, p = 10 - 2 - 3 = 5 \), and ABB’s profit is

\[\pi_2 = (5 - 2)\times3 - 7 = 2.\]

Thus, we know that there is some factory size that would just deter ABB from entering the market.