1. \textit{AccBert\_a.}

(a) Firm 1’s profit maximization problem is:

$$\max_{p_1} \pi_1(p_1, p_2) = (p_1 - 1)(1 - 0.3p_1 + 0.1p_2)$$

Its first order condition is:

$$\frac{\partial \pi}{\partial p_1} = 1.3 - 0.6p_1 + 0.1p_2 = 0$$

Solving for \( p_1 \) and exploiting the symmetry of the problem, we can get the reaction functions for firms 1 and 2:

$$p_1(p_2) = \frac{1.3 + 0.1p_2}{0.6} \quad p_2(p_1) = \frac{1.3 + 0.1p_1}{0.6}$$

When we set these equal and solve simultaneously, the Nash equilibrium is \( p_1^* = p_2^* = 2.6 \). The corresponding quantities are

$$q_1(2.6, 2.6) = q_2(2.6, 2.6) = 0.48$$ and the profits are

$$\pi_1(2.6, 2.6) = \pi_2(2.6, 2.6) = 0.768.$$

(b) Firm 1’s new second-period profit maximization problem is:

$$\max_{p_1} \pi_1(p_1, p_2) = p_1(1 - 0.3p_1 + 0.1p_2)$$

Its first order condition and reaction function are:

$$\frac{\partial \pi}{\partial p_1} = 1 - 0.3p_1 + 0.1p_2 - 0.3p_1 = 0 \Rightarrow p_1(p_2) = \frac{1 + 0.1p_2}{0.6}$$

For firm 2, the reaction function is unchanged from part (a), so solving simultaneously gives

$$p_1(p_2(p_1)) = \frac{1}{0.6} + \frac{0.1}{0.6} \frac{1.3 + 0.1p_1}{0.6} = 1.67 + 0.36 + 0.028p_1.$$
Solving for \( p_1 \) and the other variables gives \( p_1^* = 2.09, p_2^* = 2.52, \)
\( q_1(2.09, 2.52) = 0.625, q_2(2.09, 2.52) = 0.453 \). The (operating) profits are
\( \pi_1 = 2.09 \times 0.625 = 1.31 \) and
\( \pi_2 = (2.52 - 1)0.453 = 0.69 \). Net profit for firm 1 is
\( \Pi_1 = 1.31 - 0.5 = 0.81 \), so the machine is just barely worthwhile.

(c) This is a game of strategic complements, which can be seen directly from the reaction functions, which slope up in the other firm’s action. In this game, investing in the machine makes firm 1 tough, in the sense that it charges a lower price, thus doing less of a complement. Thus, this is an example of the lean-and-hungry look – firm 1 invests to be very efficient and scare firm 2.

2. *ShyNetworks2.5.2.a.* Suppose there are two networks, A and B, with user-bases \( n_A \) and \( n_B \). There are \( a \) users who prefer network A and \( b \) users who prefer network B, with \( a + b = 1 \). Let type \( a \) users have utility \( U_a = 2n_a \) if they buy platform A and \( U_a = 2n_b - 0.5 \) if they buy platform B. Let type \( b \) users have utility \( U_b = 3n_a - 0.5 \) if they buy platform A and \( U_b = 3n_b \) if they buy platform B. (So the network externality is stronger for the \( b \)-types.)

(a) Suppose \( n_A = 0 \) and \( n_B = 1 \). If an \( a \)-type switched, they would go from utility \( 2 - 0.5 = 1.5 \) to utility 0. If a \( b \)-type switched, they would go from utility 3 to utility \( 0 - 0.5 = -0.5 \). Both types would lose from switching, so this is a Nash equilibrium.

(b) Social welfare is \( W = a \times 1.5 + b \times 3 \).

(c) If \( n_A = n_B = 0.5 \), utility for \( a \)-types is \( U_a(A) = 1 \) and
\( U_a(B) = 1 - 0.5 = 0.5 \), so an \( a \)-type would not switch. The utility for \( b \)-types is \( U_b(B) = 1.5 \) and \( U_b(A) = 1.5 - 0.5 = 1 \), so they would not switch either. So yes, this is a Nash equilibrium. Social welfare is \( W = 0.5 \times 1 + 0.5 \times 1.5 = 1.25 \), but it using the
expression in part (b), it would have been $0.5 \times 1.5 + 0.5 \times 3 = 2.25$ if everyone standardized on B.

(d) Obviously the $b$-types are going to be happy with their network as it becomes larger. So the question is whether the $a$-types would stick with A. For their utility on A to be greater than switching, we need

$$U_a(A) > U_a(B) \Rightarrow 2a > 2b - 0.5 \Rightarrow 2(1 - b) > 2b - 0.5 \Rightarrow b < 0.625$$

So $b = 0.625$ is the largest $b$ population such that a non-standardized equilibrium still exists?