1. *M1-1.a.* The monopoly profit maximization problem is:

\[ \max_{q} \pi(q) = (100 - q - 10)q \]

The first order condition (equivalent to setting marginal revenue equal to marginal cost) is

\[ \frac{\partial \pi}{\partial q} = 90 - q - q = 0 \Rightarrow q = 45 \]

so the price is \( p = 100 - 45 = 55 \). The Lerner Index is

\[ \frac{p - MC}{p} = \frac{55 - 10}{55} = 0.82 \]

which is to say that 82% of the price is monopoly markup.

Under perfect competition, the price would be 10, and quantity would be \( 10 = 100 - q \Rightarrow q = 90 \). Thus, the deadweight loss is a triangle with base of 90 - 45 and a height of 55 - 10, which has area:

\[ DWL = \frac{1}{2}(90 - 45)(55 - 10) = 1012.5 \]
2. *KmartWalMart*. In this case, Wal-Mart has $MC_W = 0.7$ while Kmart has $MC_K = 1$. So to begin, we find the Cournot reaction function for a firm 1 with marginal cost $c$ facing a rival firm 2:

$$\max_{q_1} = (500 - 4(q_1 + q_2))q_1 - cq_1$$

\text{FOC: } \begin{align*}
500 - 4q_1 - 4q_2 - 4q_1 &= c \\
500 - 8q_1 - 4q_2 &= c \\
8q_1 &= 500 - c - 4q_2 \\
q_1(q_2) &= 62.5 - \frac{c}{8} - \frac{q_2}{2}
\end{align*}

Note the above is true for any type of firm, since firms don’t directly care about opponents’ costs, but only indirectly through the effect on $q_2$. If we now plug in the marginal costs given in the problem, we get

$$q_K(q_W) = 62.375 - \frac{q_W}{2} \quad q_W(q_K) = 62.4125 - \frac{q_K}{2}$$

Now solving simultaneously to find a Cournot-Nash equilibrium:

$$q_K = 62.375 - \frac{62.4125 - \frac{q_K}{2}}{2}$$

$$q_K = 62.375 - 31.206 + \frac{q_K}{4}$$

$$q_K \left(1 - \frac{1}{4}\right) = 31.169$$

$$q_K = 41.559$$

And plugging this back into the $q_W$ function gives

$$q_W = 62.4125 - \frac{41.559}{2} = 41.633$$

At these outputs, the price is $p(41.559 + 41.633) = 167.23$, Kmart’s profit is $(167.23 - 1)41.559 = 6,908.44$ and Wal-Mart’s profit is $(167.23 - 0.7)41.633 = 6,933.14$. The market shares are $41.559/83.192 = 49.96\%$ for Kmart and $41.633/83.192 = 50.04\%$ for Wal-Mart.

(Next time round, this problem will be improved by reducing the demand intercept from 500 to 2, which will make the scaling better and give larger market share differences.)
3. **M2-1_a.** The weighted average can be rewritten to make it more similar to the usual problem:

\[ g_i = p_i q_i - (1 - \sigma) 10 q_i \]

Essentially, the firms are underweighting costs. The maximization problem for firm 1 is

\[ \max_{q_1} g_1 = (100 - q_1 - q_2) q_1 - (1 - \sigma) 10 q_1 \]

The best response function for firm 1 is just the FOC for a payoff maximum (note it’s not a profit maximum in this case):

\[ \frac{\partial g_1}{\partial q_1} = 100 - q_1 - q_2 - q_1 - (1 - \sigma) 10 = 0 \Rightarrow q_1(q_2) = \frac{100 - q_2 - (1 - \sigma) 10}{2} \]

We already know that problems of this form have a solution where quantity is \( \frac{a - c}{3b} \) and profit is \( \frac{(a-c)^2}{9b} \). Here \( a = 100, b = 1, \) and \( c = (1 - \sigma) 10, \) so the answers are

\[ q_1^* = \frac{100 - (1 - \sigma) 10}{3} \quad \pi_i^* = \frac{(100 - (1 - \sigma) 10)^2}{9} \]

If we draw a best response diagram and compare the standard case \( \sigma = 0 \) with this case, we can see that the effect of \( \sigma \) is to shift the reaction functions to the right (up for firm 2), increasing the total amount of Cournot equilibrium quantity. Of course this actually decreases the correctly-calculated profit.