1. **Bertrand Collusion.** There are two firms which are differentiated Bertrand competitors. They have demand curves:

\[ q_1 = 1 - 0.3p_1 + 0.1p_2 \]
\[ q_2 = 1 - 0.3p_2 + 0.1p_1 \]

The firms have identical, constant marginal costs of $1 per unit.

(a) What is the differentiated Bertrand equilibrium profit?

(b) If the two firms colluded, what would be the profit of each firm? (You can confine attention to actions that set the prices equal since the firms are symmetric.)

(c) If one firm cheated on this collusive agreement, what profit would it make?

(d) Suppose the game were repeated. For what discount rates could the firms sustain a tacitly collusive trigger strategy equilibrium?

2. **Chocolate.** Two firms supply cacao at a wholesale market in Trinidad. Firm 1 has always had lower costs than firm 2, reflected by constant marginal costs \( c_1 < c_2 \). Market demand for cocoa is \( p = a - Q \), where \( Q = q_1 + q_2 \).

(a) If these firms behaved like perfect competitors, what would be each firm’s output, market price, and market quantity?

(b) Now suppose the two firms behave as Cournot competitors. What is the Cournot equilibrium quantity produced by both firms, the market price, and the equilibrium profit of firm 1?
(c) If firm 1 could move first, followed by firm 2, what would be the Stackelberg equilibrium quantity produced by both firms, the market price, and the equilibrium profit of firm 1?

**Review problem with answer:**

3. **Shy6.8.3.** Consider a 3-firm version of the Stackelberg game. Assume that market inverse demand is given by $p = 120 - Q$, and suppose that there are three firms that set their output sequentially: firm 1 sets $q_1$ in period 1, firm 2 sets $q_2$ in period 2, and firm 3 sets $q_3$ in period 3. Then, firms sell their output and collect their profits. Solve for the sequential-moves (i.e. Stackelberg) equilibrium. Make sure that you solve for the output level of each firm, and for the market price.

3. **Shy6.8.3_a.** We have to work backwards, starting with stage 3. At that point, $q_1$ and $q_2$ are given, and firm 3 maximizes:

$$\max_{q_3} \pi_3 = (120 - q_1 - q_2 - q_3)q_3$$

This gives first order condition

$$\frac{\partial \pi_3}{\partial q_3} = 120 - q_1 - q_2 - q_3 - q_3 = 0$$

and reaction function

$$q_3(q_1, q_2) = 60 - \frac{1}{2}q_1 - \frac{1}{2}q_2$$

In Stage 2, firm 2 maximizes its profits, expecting firm 3 to behave as we just derived:

$$\max_{q_2} \pi_2 = (120 - q_1 - q_2 - q_3(q_1, q_2))q_3 = (60 - \frac{1}{2}q_1 - \frac{1}{2}q_2)q_2$$

This gives first order condition

$$\frac{\partial \pi_2}{\partial q_2} = 60 - \frac{1}{2}q_1 - \frac{1}{2}q_2 - \frac{1}{2}q_2 = 0$$
and reaction function

\[ q_2(q_1) = 60 - \frac{1}{2} q_1 \]

In Stage 1, firm 1 knows that firm 2 will play as above. It also knows that firm 3 will go on to react according to

\[ q_3(q_1, q_2(q_1)) = 60 - 0.5q_1 - 30 + 0.25q_1 = 30 - 0.25q_1 \]

Thus, firm 1 maximizes

\[ \max_{q_2} \pi_2 = (120 - q_1 - (60 - 0.5q_1) - (30 - 0.25q_1))q_1 = (30 - 0.25q_1)q_1 \]

This gives first order condition

\[ \frac{\partial \pi_1}{\partial q_1} = 30 - 0.25q_1 - 0.25q_1 = 0 \]

and optimal Stackelberg leader quantity

\[ q_1 = 60 \]

Plugging this leader quantity back into the other reaction functions, we see that the other two firms produce

\[ q_2 = 30 \quad q_3 = 15 \]

The total market quantity is thus 105, and the market price is

\[ p = 120 - 105 = 15. \]