Toll Competition Among Congested Roads
by Engel, Fisher, and Galetovic

Numerical example from pg. 7

Let $B(Q) = 1 - q_1 - q_2$

Let costs be $c_1(q_1) = 0.3q_1$ and $c_2(q_2) = 0.5q_2$

Then market “flow” onto roads requires:

\begin{align*}
1 - (q_1 + q_2) &= 0.3q_1 + p_1 \\
1 - (q_1 + q_2) &= 0.5q_2 + p_2
\end{align*}

Give demand functions

\begin{align*}
q_1(p_1, p_2) &= 0.53 - 1.58p_1 + 1.05p_2 \\
q_2(p_2, p_1) &= 0.32 - 1.37p_2 + 1.05p_1
\end{align*}
Oligopoly tolls too high – part 2

So each firm maximizes profits $\Pi_i = p_i q_i$
– notes costs are for consumers

Reaction functions are

$$p_1(-1.58) + 0.53 - 1.58p_1 + 1.05p_2 = 0 \Rightarrow p_1(p_2) = \frac{0.53 + 1.05p_2}{3.16}$$
$$p_2(-1.37) + 0.32 - 1.37p_2 + 1.05p_1 = 0 \Rightarrow p_2(p_1) = \frac{0.32 + 1.05p_1}{2.74}$$

so strategic complements

Optima: $p_1 = 0.24$ $p_2 = 0.06$ $q_1 = 0.21$ $q_2 = 0.49$

The externality on road 1 is $q_1c'_1 = 0.21 \cdot 0.3 = 0.06$. The externality on road 2 is $q_2c'_1 = 0.49 \cdot 0.5 = 0.245$.

Thus, the toll is too high, congestion is too low.
Note, *no* product differentiation.
Profits are $\Pi_1 = 0.24 \cdot 0.21 = 0.05$
$\Pi_2 = 0.06 \cdot 0.49 = 0.0294$