1. *M2-I.a.* The monopoly profit maximization problem is:

$$\max_q \pi(q) = (100 - q - 10)q$$

The first order condition (equivalent to setting marginal revenue equal to marginal cost) is

$$\frac{\partial \pi}{\partial q} = 90 - q - q = 0 \Rightarrow q = 45$$

so the price is $p = 100 - 45 = 55$. The Lerner Index is

$$\frac{p - MC}{p} = \frac{55 - 10}{55} = 0.82$$

which is to say that 82% of the price is monopoly markup.

Under perfect competition, the price would be 10, and quantity would be $10 = 100 - q \Rightarrow q = 90$. Thus, the deadweight loss is a triangle with base of $90 - 45$ and a height of $55 - 10$, which has area:

$$DWL = \frac{1}{2} (90 - 45)(55 - 10) = 1012.5$$
2. \textit{KmartWalMart}. In this case, Wal-Mart has $MC_W = 0.7$ while
Kmart has $MC_K = 1$. So to begin, we find the Cournot reaction
function for a firm 1 with marginal cost $c$ facing a rival firm 2:

$$\max_{q_1} = (2 - 4(q_1 + q_2))q_1 - cq_1$$

FOC: \begin{align*}
2 - 4q_1 - 4q_2 - 4q_1 &= c \\
2 - 8q_1 - 4q_2 &= c \\
8q_1 &= 2 - c - 4q_2 \\
q_1(q_2) &= 0.25 - \frac{c}{8} - \frac{q_2}{2}
\end{align*}

Note the above is true for any type of firm, since firms don’t directly
care about opponents’ costs, but only indirectly through the effect
on $q_2$. If we now plug in the marginal costs given in the problem,
we get

$$q_K(q_W) = 0.125 - \frac{q_W}{2} \quad q_W(q_K) = 0.1625 - \frac{q_K}{2}$$

Now solving simultaneously to find a Cournot-Nash equilibrium:

$$q_K = 0.125 - \frac{0.1625 - \frac{q_K}{2}}{2}$$

$$q_K = 0.125 - 0.08125 + \frac{q_K}{4}$$

$$q_K \left(1 - \frac{1}{4}\right) = 0.04375$$

$$q_K = 0.058$$

And plugging this back into the $q_W$ function gives

$$q_W = 0.1625 - \frac{0.058333}{2} = 0.13$$

At these outputs, the price is $p(0.13 + 0.058) = 1.23$, Kmart's
profit is $(1.23 - 1)0.058333 = 0.0136$ and Wal-Mart's profit is
$(1.23 - 0.7)0.13 = 0.0689$. The market shares are $0.058/0.188 = 30.8\%$ for Kmart and $0.13/0.188 = 69\%$ for Wal-Mart.