Single Variable Calculus Review

1. Find the limit.
   
   (a) \( \lim_{x \to 0} \cos(x + \sin x) \)
   
   (b) \( \lim_{x \to 3} \frac{x^2 - 9}{x^2 + 2x - 3} \)
   
   (c) \( \lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} \)
   
   (d) \( \lim_{x \to 1^+} \frac{x^2 - 9}{x^2 + 2x - 3} \)

2. Differentiate.

   (a) \( y = (x^4 - 3x^2 + 5)^3 \)
   
   (b) \( y = \cos(\tan \theta) \)
   
   (c) \( z = \sqrt{x} + \frac{1}{\sqrt{x^2}} \)
   
   (d) \( y = \frac{1}{1-t^2} \)
   
   (e) \( h(\theta) = \frac{\sec 2\theta}{1 + \tan \theta} \)
   
   (f) \( w = \sin(\cos z) \)
   
   (g) \( f(t) = t^2 \ln t \)
   
   (h) \( y = \frac{e^t}{1+e^t} \)

3. Find an equation of the line tangent to the curve \( y = 4\sin^2 x \) at the point \( (\pi/6, 1) \).

4. Find the local and absolute extreme values of \( f(x) = x^3 - 6x^2 + 9x + 1 \) on the interval \([2, 4]\).

5. A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at \$12\), average attendance at a game has been 11,000. A market survey indicates that for each dollar the ticket price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price to maximize their revenue from ticket sales?

6. Evaluate the integral.

   (a) \( \int_1^2 (8x^3 + 3x^2) \, dx \)
   
   (b) \( \int_0^1 y(y^2 + 1)^5 \, dy \)
   
   (c) \( \int_{1/12}^{1/6} \sin(3\pi t) \, dt \)
   
   (d) \( \int_0^{\pi/8} \sec 2\theta \tan 2\theta \, d\theta \)
   
   (e) \( \int_2^5 \frac{dv}{1+v^2} \)

7. Find the area of the region that lies under the curve \( y = x\sqrt{x} \), \( 0 \leq x \leq 4 \).

8. Evaluate the integral.

   (a) \( \int_0^1 \frac{e^x}{1+e^x} \, dx \)
   
   (b) \( \int \frac{x+1}{x^2+2x} \, dx \)
   
   (c) \( \int \cos^2 x \sin^3 x \, dx \)
   
   (d) \( \int x e^x \, dx \)
   
   (e) \( \int \frac{\sqrt{x^2-1}}{x} \, dx \)
   (a) \( f(t) = 2t\sqrt{t^2 + 1} \)
   (b) \( y = \csc(1 + x^2) \)
   (c) \( y = \cot(3x^2 + 5) \)
   (d) \( y = \ln(x^2e^x) \)
   (e) \( y = (\arcsin 2x)^2 \)
   (f) \( y = \arctan(\arcsin \sqrt{x}) \)
   (g) \( y = \ln(\sec^2 x) \)

10. Find the points on the ellipse \( x^2 + 2y^2 = 1 \) where the tangent line has slope 1.

11. A particle moves on a vertical line so that its coordinate at time \( t \) is \( y = t^3 - 12t + 3, \ t \geq 0 \).
   (a) Find an expression that describes the velocity of the particle as a function of time.
   (b) Find an expression that describes the acceleration of the particle as a function of time.
   (c) When is the particle moving upward and when is it moving downward?
   (d) When is the particle speeding up? When is it slowing down?

12. The volume of a cube is increasing at a rate of 10 cm\(^3\)/min. How fast is the surface area increasing when the length of an edge is 30 cm?

13. Evaluate the integral.
   (a) \( \int_{\pi/6}^{\pi/3} v^2 \cos v^3 \, dv \)
   (b) \( \int_{0}^{1} (1 - x)^9 \, dx \)
   (c) \( \int_{0}^{\pi/4} (1 + \tan t)^3 \sec^2 t \, dt \)
   (d) \( \int \frac{\cos(\ln x)}{x} \, dx \)

14. Find the area of the region bounded by the curves \( y = x^2 \) and \( y = 4x - x^2 \).

15. A particle moves along a line with velocity function \( v(t) = t^2 - t \), where \( v \) is measured in meters per second.
   (a) Find an expression that describes the displacement of the particle as a function of time.
   (b) Find the distance traveled by the particle during the time interval \([0, 5]\).

16. Evaluate the integral.
   (a) \( \int \frac{1}{y^2 - 4y - 12} \, dy \)
   (b) \( \int x^{3/2} \ln x \, dx \)
   (c) \( \int \frac{x - 1}{x^2 + 2x} \, dx \)